A finite element thermal stress model to compute the thermomechanical state of the solidifying shell during continuous casting of steel in a square billet casting mould has been applied to investigate longitudinal cracks. A two-dimensional thermoelastoviscoelastic analysis was carried out within a horizontal slice of the solidifying strand which moves vertically within and just below the mould. The model calculates the temperature distributions, the stresses, the strains in the solidifying shell, and the intermittent air gap between the casting mould and the solidifying strand. Model predictions were verified with both an analytical solution and a plant trial. The model was then applied to study the effect of mould corner radius on longitudinal crack formation for casting in a typical 0·75% Cr steel billet with both oil and mould powder lubrication. With this inadequate linear taper, a gap forms between the shell and the mould in the corner region. As the corner radius of the billet increases from 4 to 15 mm, this gap spreads further around the corner towards the centre of the strand and becomes larger. This leads to more temperature non-uniformity around the billet perimeter as solidification proceeds.

Longitudinal corner surface cracks are predicted to form only in the large corner radius billet, owing to tension in the hotter and thinner shell along the corner during solidification in the mould. Off corner internal cracks form more readily in the small corner radius billet. They are caused by bulging below the mould, which bends the thin, weak shell around the corner, creating tensile strain on the solidification front where these longitudinal cracks are ultimately observed.

INTRODUCTION

During the continuous casting of steel billets, the corner regions of the cast section often experience local thinning. This phenomenon, sometimes referred to as ‘re-entrant corners’, results from the complex behaviour of the air gap, which forms between the mould and the solidifying shell in the corner region. This common occurrence can lead to problems such as longitudinal cracks near the billet corner, especially at high casting speed.1–3 In extreme cases, the corners may be so thin that a breakout occurs, even though the average shell thickness is easily large enough to withstand the ferrostatic pressure at the mould exit.

Two decades of operating experience have shown that reducing the corner radius from 12–16 mm to 3 or 4 mm is beneficial in reducing longitudinal corner cracking.4 In addition to lessening crack frequency, decreasing the corner radius also tends to move the crack location from the corner itself to the off corner region. Unfortunately, billets with sharp edges tend to ‘fold over’ during the rolling process.5 Therefore, mould designers struggle to satisfy these two conflicting requirements. A better way to solve the longitudinal corner crack problems is desirable. An important step towards this end is the achievement of an accurate, quantitative understanding of the crack formation mechanisms. This understanding would aid mould design optimisation, especially for high speed casting.

Over the years, many mathematical models have been developed to help to understand the origin of defects in complex processes such as continuous casting.6–11 However, quantitative understanding of the re-entrant corner phenomenon of the solidifying shell in the billet mould has received relatively little attention. Furthermore, the effect of the billet mould corner radius on the temperature, corner gap, and stress development has not been studied.

In the present work, a thermal–elastic–plastic–creep finite element model has been developed to study the thermal–mechanical behaviour of the solidifying shell in and just below a billet mould. The model was validated with plant measurements including solid shell thickness and mould thermocouple temperatures. The model was then applied to the re-entrant corner phenomenon to investigate the influence of corner radius on longitudinal crack formation.

PREVIOUS WORK

Longitudinal cracks

Longitudinal cracks are one of the most common mould related quality problems encountered in billet casting. They are associated with hot tearing close to the solidification front,2 and are manifested in at least two different forms: longitudinal corner cracks and ‘off corner internal cracks’.1,2,12

Longitudinal corner cracks run along the surface near the exact corner of the billet and are usually 1–2 mm in depth,12 as shown in Fig. 1a. Although several studies suggest that longitudinal corner cracks are related to the rhomboid condition of the billet,12,14–17 these cracks also occur in the absence of rhomboidity, as a result of improper corner radius12,18 or mould distortion and wear.14,15 Aketa and Ushijima18 observed that with a large corner radius, the longitudinal corner cracks appear along the corner, while with smaller radii, these surface cracks form more frequently at the off corner region. They suggested that the optimal corner radius to minimise longitudinal crack formation should be one-tenth of the section size.2 However, Samarasekera and Brimacombe12 believed that the modern trend of smaller corner radii such as 3 or 4 mm may solve the longitudinal corner cracking problem, but at the expense of creating more off corner cracks. Mori17 observed that the incidence of longitudinal corner cracks increases with the time that a mould is in service during a campaign. He suggested that overall reverse of taper may be an important
Mathematical stress models

During continuous casting, solidification of the steel shell in the mould region involves many complex phenomena such as fluid flow, interaction of shrinkage of the shell and ferrostatic pressure, which leads to intermittent contact with the mould, and interaction of interfacial heat transfer with air gap formation. Over the years, many mathematical models have investigated the thermal and mechanical behaviour of the solidifying shell with air gap formation in the continuous casting of steel in a billet mould.\textsuperscript{6–11}

Grill et al.\textsuperscript{6} applied an elastic–plastic model of the billet strand to study its thermomechanical behaviour and to explain internal crack formation. They calculated the heat transfer coefficient in the corner region and were able to predict corner cracks in the billet by coupling heat flow to the air gap computed from stress analysis. The model was improved later by Sorimachi and Brimacombe\textsuperscript{7} with better material property data. They observed that internal cracks could be caused by surface reheating below the mould.

Kristiansson and Zetterlund\textsuperscript{8,9} simulated billet casting using a stepwise coupled two-dimensional thermal and mechanical model, which also calculated the size of the shell–mould gap around each portion of the strand periphery at each time. The model was applied to investigate the formation of longitudinal subsurface cracks in the solidifying shell. They suggested that large air gaps, which may form owing to wear or misalignment of the mould, cause large strains in the solidifying shell and a high risk of cracking.

Kelly et al.\textsuperscript{10} developed a coupled two-dimensional axisymmetric thermomechanical model for steel shell behaviour in round billet casting moulds using a combination of models FIDAP and NIKED2D. Their model was fully coupled through the interface gap, included mould distortion, and assumed elastic–plastic mechanical behaviour. Their results suggested that thermal shrinkage associated with the phase change from δ ferrite to austenite in 0·1%C steel accounts for the decreased heat transfer observed in this alloy as well as its susceptibility to cracking.

Tseng et al.\textsuperscript{11} calculated billet temperature fields using a temperature recovery solidification method, followed by an uncoupled stress analysis with plane strain in the MARC model. They interpreted the results to obtain qualitative ideas about possible billet defects.

Ohnaka and Yashima\textsuperscript{12} studied the effect of mould taper and mould corner radii on the temperature and stress fields in slab casting using an elastoplastic model, which considered the ferrostatic pressure, mould taper, and interaction between the solidifying shell and mould. This model demonstrated that shell deformation owing to thermal stress and ferrostatic pressure changes the shell–mould thermal resistance, resulting in tensile stress near the slab corner, which may cause longitudinal cracks. They also suggested that a larger mould corner radius should decrease the interfacial gap thickness and tensile stress in the shell and thereby help to prevent cracks.

In the present work, a thermoelasoviscoplastic finite element model has been developed to simulate temperature and stress in a transverse slice through the solidifying shell of a typical billet caster. The evolution of the air gap has been calculated from the deformation of the strand and the tapered and distorted mould. Its coupled effect on the temperature distribution has been taken into account with a distance dependent heat transfer coefficient between the mould and strand. The accuracy of the two-dimensional (2D) slice model formulation in this analysis has also been
investigated through comparison with both an analytical solution and measurements from a plant trial. Finally, the model has been applied to the specific problem of how the corner radius of the mould affects the thermal, deformation, and stress fields of a low carbon steel billet continuously cast using both oil lubricant and mould powder practices. The implications for longitudinal crack formation are discussed.

PLANT TRIALS
Caster details and nominal operating practice
A plant trial was conducted at POSCO, Pohang works, South Korea, relating to a 120 mm square section of 0.04%C steel continuously cast at 2.2 m min⁻¹. The mould was manufactured from relatively pure, deoxidised high purity (DHP) copper with a wall thickness of 6 mm, a corner radius of 4 mm, and a single ‘linear’ taper of 0.75%/m. Other operating parameters and mould geometry details are provided in Table 1.

Mould temperature measurement
The mould tube was instrumented with 12 K type thermocouples on the inside radius face as shown in Fig. 2. They were arranged in three columns along the centreline and ±45 mm from the centreline, and in four rows located at 120, 170, 400, and 700 mm below the top of the 800 mm length mould. The thermocouples were embedded in the mould wall to a depth of 3 mm from the hot face. The mould water temperature increase was not recorded at the time, but is estimated to be 30 K based on recent measurements for the same conditions.

Solid shell measurement
To investigate solid shell growth, FeS tracer was suddenly added into the liquid pool during steady state casting. Because FeS cannot penetrate the solid shell, the position of the solid shell front at that instant can be clearly recognised after casting using a sulphur print.

MATHEMATICAL MODEL DESCRIPTION
To investigate the thermomechanical behaviour of the continuous cast billet and mould, a 2D transient thermoelasto-viscoplastic finite element model (AMEC2D)²⁰–²² has been developed. This model tracks the thermal and mechanical behaviour of a transverse slice through the continuously cast strand as it moves down through the caster. The model includes separate finite element models of heat transfer and stress generation that are stepwise coupled through the size and properties of the interfacial gap. Stresses arise primarily as a result of thermal strains, while heat transfer across the gap depends on the amount of shrinkage of the solidifying shell. During each step of the analysis, the temperature fields of the mould and strand are calculated simultaneously, extrapolating from the previous step, neglecting axial conduction. Then, the stress analysis calculates deformation of the strand, stress, and the air gap size. Iteration continues until the heat transfer coefficient determined from the calculated gap is converged.

Microsegregation analysis
Generally, the solidification of steel during continuous casting does not exactly follow the path of the equilibrium binary Fe–C phase diagram owing to the rapid cooling and microsegregation of other solute elements. To determine the variation of liquid, δ-Fe, and γ-Fe fractions with temperature, the microsegregation of solute elements of steel was analysed using the direct finite difference method of Kim²³ and Ueshima et al.²⁴ as described elsewhere.²⁵ Figure 3 shows the calculated liquid, δ-Fe, and γ-Fe fractions as a function of temperature during solidification of the low carbon steel grade used in the plant trial (Fe-0.04C-0.2Si-0.25Mn-0.010P-0.015S, wt-%) and the corresponding thermal linear expansion (TLE) function used in the present study. These results were used to determine the thermophysical properties of the steel given below.
Heat flow analysis

The heat flow model solves the 2D transient heat conduction equation for the temperature distribution in the solidifying shell. The effects of solidification and solid state phase transformation on the heat flow are incorporated through a temperature dependent enthalpy function as shown in Fig. 4. This figure also shows the temperature dependent conductivity function.

The following assumptions are used in this calculation:
(i) the incoming metal temperature, liquid level, and casting speed are constant and axial heat conduction is ignored
(ii) mould oscillation and friction between the shell and the mould are neglected
(iii) the effect of convective heat flow in the liquid region is taken into account using the effective thermal conductivity $k_{eff}$ for molten steel

$$
\frac{1}{k_{eff}} = \frac{1}{h} + \frac{1}{k_{shell}} + \frac{1}{d_{gap}k_{flux}}
$$

where $f_s$ is the solid fraction.

Oil casting interface heat transfer

Heat extraction from the solid shell surface in the mould is primarily controlled by heat conduction across the interface between the mould and the solidifying shell. This is modelled as an internal boundary condition, using the interfacial heat transfer coefficient $h_c$ as a function of air gap thickness and surface temperature of the strand, according to the relationship of Kelly et al.\textsuperscript{10}

$$
h_c = h_{rad} + \frac{1}{R_T} = h_{rad} + \frac{k_{flux}}{d_{gap}} + 1
$$

where $R_T$ is the thermal resistance, $K_g$ is the thermal conductivity of the gap medium (assumed to be 100% air in the present study), given in Table 2, $d_{gap}$ is the thickness of the gap, and $h_{rad}$ is the heat transfer coefficient for radiative heat flow when an air gap exists between the strand and the mould such that

$$
h_{rad} = \frac{\varepsilon s c}{(T_f - T_m)(T_f^2 + T_m^2)}
$$

where $\varepsilon s$ is the Stefan–Boltzmann constant, $T_f$ is the shell surface temperature, and $T_m$ is the mould hot face temperature. The average emissivity $\varepsilon$ of the shell and mould surface is assumed to be 0.8.\textsuperscript{27} If the value of $h_c$ computed from equation (2) exceeds the value associated with direct contact, it is truncated to that value. The value of $h_c$ for direct contact is taken to be 2500 W m\(^{-2}\) K\(^{-1}\), which represents a minimum contact resistance or average gap effects of solidification and solid state radiative heat flow when an air gap exists between the strand and the mould for various air gap sizes and given surface temperatures.

Powder casting interface heat transfer

To study the effect of using mould powder as a lubricant, simulations were also performed using the following expression for thermal resistance between the solidifying shell surface and the mould, consisting of four terms

$$
R_T = \frac{1}{h_{m}} + \frac{d_{m}}{K_g} + \frac{d_{flux}}{K_{flux}} + \frac{1}{h_{shell}}
$$

The first thermal resistance (first term in equation (4)) is the contact resistance between the mould wall surface and the mould flux, where $h_m$ is the contact heat transfer coefficient set to 2500 W m\(^{-2}\) K\(^{-1}\). The second resistance is conduction through the air gap, which is the same as calculated for oil casting. The third resistance is conduction through the mould flux film, with a thermal conductivity $K_{flux}$ of 1.0 W m\(^{-1}\) K\(^{-1}\).\textsuperscript{21} The thickness of the mould flux layer $d_{flux}$ is assumed to be 0.1 mm.\textsuperscript{26} The final term is the contact resistance between the mould flux and the strand surface, where the heat transfer coefficient $h_{shell}$ depends

Table 2 Conductivity of gap medium (air) with temperature

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>Conductivity, W m(^{-1}) K(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.032</td>
</tr>
<tr>
<td>400</td>
<td>0.039</td>
</tr>
<tr>
<td>600</td>
<td>0.045</td>
</tr>
<tr>
<td>800</td>
<td>0.051</td>
</tr>
<tr>
<td>1000</td>
<td>0.057</td>
</tr>
<tr>
<td>1200</td>
<td>0.063</td>
</tr>
<tr>
<td>1400</td>
<td>0.068</td>
</tr>
</tbody>
</table>
greatly on temperature, because of the large change in viscosity of the mould flux over the temperature range of the strand surface. The temperature dependency of $h_{\text{shell}}$ is given in Table 3.$^{30}$

**Spray cooling**

To investigate bulging of the billet below the mould, thermal calculations were extended to 200 mm below the mould exit, assuming a value of 500 W m$^{-2}$ K$^{-1}$ for the heat transfer coefficient at the billet surface and ambient temperature of 30°C. This value was chosen to represent a typical spray cooling coefficient, which ranges from 200 to 600 W m$^{-2}$ K$^{-1}$ in the literature.$^{30}$

**Mould temperature**

Temperature in the mould was assumed to be steady within each time step and slice. It was calculated in AMEC2D by applying the water heat transfer coefficient to the cold face of the mould based on the correlation of Dittus and Boelter.$^{31}$ This analysis ignores axial heat conduction. Thus, a second model, CON1D,$^{32}$ was applied to validate the thermal model. This model takes into account axial heat conduction in the mould, so gives more accurate mould temperature predictions than AMEC2D.

**Stress analysis**

The stresses and strain distributions associated with temperature change in the transverse slice of the solidifying shell are calculated by solving the standard equilibrium, stress–strain, and small strain displacement equations. The slice is assumed to be in a plane strain condition, in which strain along the casting direction is neglected. The temperatures calculated by the thermal model are input to the incremental thermal stress model.

**Mould taper and distortion**

Mould distortion due to thermal expansion, which is added to the mould taper, to define the mould wall position, is calculated from

$$\Delta x_{\text{mould}} = \frac{x_{\text{mould}}}{\text{mould width}} \left( \frac{T_{\text{cold}} + T_{\text{melt}} - T_{\text{ref}}} {2} \right)$$

(5)

where $x_{\text{mould}}$ is the mould thermal linear expansion coefficient (1·6 × 10$^{-3}$ K$^{-1}$), $T_{\text{cold}}$ is the mould cold face temperature (°C), $T_{\text{melt}}$ is the mould hot face temperature (°C), and $T_{\text{ref}}$ is the average mould temperature at the meniscus (°C).

For equation (5), the mould temperature is based on the results of the CON1D model,$^{33}$ which matches well with the measured temperature. Figure 6 shows profiles of the mould distortion, 0·75%/m linear profile of the mould taper, and the actual mould wall shape adopted in the present work as the wall boundary condition.

**Thermal strain**

Thermal strain arises from the volume changes caused by changing temperature and phase transformation. This was calculated from the temperature determined in the heat transfer analysis and the thermal linear expansion of steel (TLE), which can be determined in turn from the phase fractions found by microsegregation analysis and the specific volume $V$ of each phase of the steel

$$\text{TLE} = \left( \frac{V}{V_{\text{ref}}} - 1 \right)^{1/3}$$

(6)

$$V = (f_f V_f + f_c V_c)$$

(7)

where $V_{\text{ref}}$ is the specific volume at the reference temperature, and $f_f$, $f_c$, and $f_l$ are fractions of $\delta$, $\gamma$, and liquid phase, respectively. The reference temperature is chosen to correspond with the solid fraction of 0·8. The specific volume of the various phases is given in Table 4, and were obtained from Wray.$^{34}$

**Effective plastic strain and flow stress in carbon steel**

At higher temperatures, it is important to stress development during solidification, inelastic strain from plasticity and creep is also important. The following constitutive equation proposed by Han and co-workers$^{35–37}$ is used to relate the flow stress of $\delta$ and $\gamma$ phases at various temperatures $T$ and strain rates $\dot{\varepsilon}_p$

$$\dot{\varepsilon}_p = A \exp \left(-Q/R T \right) \sinh (\beta K)^{1/m}$$

(8)

$$\sigma = K \dot{\varepsilon}_p^{m}$$

(9)

where $A$ and $\beta$ are constants, $Q$ and $R$ are the activation energy for deformation and the gas constant, respectively, $m$ is the strain rate sensitivity, $K$ is the strength coefficient, $n$ is the strain hardening exponent, $\sigma$ is the flow stress, and $\dot{\varepsilon}_p$ is the effective plastic strain. Table 5 gives the parameters in the above equation for $\delta$ ferrite and $\gamma$ austenite phases of steel. The total strain rate is thus composed of this viscoplastic strain rate together with the thermal and elastic strain rates.

**Table 3** Temperature dependence of heat transfer coefficient between mould flux and strand surface

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>$h_{\text{shell}}$, W m$^{-2}$ K$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mould flux crystalline temperature, 1030°C</td>
<td>1000</td>
</tr>
<tr>
<td>Mould flux softening temperature, 1150°C</td>
<td>2000</td>
</tr>
<tr>
<td>Metal solidus temperature, 1511°C</td>
<td>10 000</td>
</tr>
<tr>
<td>Metal liquidus temperature, 1529°C</td>
<td>20 000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase</th>
<th>Specific volume, cm$^3$ g$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0·1234 + [9·28 × 10$^{-4}$] (T - 201)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0·1255 + [9·45 × 10$^{-4}$] (T - 201) + (7·688 × 10$^{-6}$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase</th>
<th>Specific volume, cm$^3$ g$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0·1234 + [9·28 × 10$^{-4}$] (T - 201)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0·1255 + [9·45 × 10$^{-4}$] (T - 201) + (7·688 × 10$^{-6}$)</td>
</tr>
</tbody>
</table>

**Table 5** Parameters for constitutive equation

<table>
<thead>
<tr>
<th>Phase</th>
<th>$A$, s$^{-1}$</th>
<th>$B$, MPa$^{-1}$</th>
<th>$Q$, kJ mol$^{-1}$</th>
<th>$m$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>6·754 × 10$^6$</td>
<td>0·0933</td>
<td>216·9</td>
<td>0·1028</td>
<td>0·0079</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1·192 × 10$^{10}$</td>
<td>0·0381</td>
<td>373·4</td>
<td>0·2363</td>
<td>0·2190</td>
</tr>
</tbody>
</table>
Elastic modulus
The elastic modulus of steel decreases significantly with increasing temperature. There is still uncertainty concerning the best value of $E$ at high temperatures. The following expression of Kinoshita et al.\textsuperscript{38} is used in the present work:

$$E = 1.38 \times 10^{-2} T^2 - 225.6 T + 3.146 \times 10^3 \text{ (kg cm}^{-2}\text{)}$$

(10)

Treatment of liquid
Since elements may be liquid, solid, or mushy, and the volume of liquid in the domain may vary, special care is needed to handle the liquid region. In the present model, negligible ($0.5 \times 10^{-4} \text{ MPa}$) stiffness is assigned to those Gaussian integration points whose temperature is above the coherence temperature, assumed to correspond to a solid fraction of 0.7. In addition, thermal expansion is assumed to be zero for temperatures corresponding to a solid fraction of 0.8 or above.

Solid shell–mould contact
Interaction between the shell and the mould affects not only the loading on the exterior position of the shell, but also influences the heat transfer significantly. A contact algorithm is applied to restrain the shell elements from penetrating the mould,\textsuperscript{39} whose position is defined in Fig. 6. At each iteration, such penetrations are evaluated, a new global matrix is generated, and stresses are resolved. To achieve convergence, the penetration parameter is set to 5.0, and the friction coefficient to 0.2.

Ferrostatic pressure and bulging
Ferrostatic pressure from the vertical gravity force on the liquid pushes the inside surface of the solidifying shell towards the mould walls, and greatly affects gap size and mould heat transfer. It increases in proportion to the volume of liquid in the domain. Since elements may be liquid, solid, or mushy, and the volume of liquid in the domain may vary, special care is needed to handle the liquid region. In the present model, negligible ($0.5 \times 10^{-4} \text{ MPa}$) stiffness is assigned to those Gaussian integration points whose temperature is above the coherence temperature, assumed to correspond to a solid fraction of 0.7. In addition, thermal expansion is assumed to be zero for temperatures corresponding to a solid fraction of 0.8 or above.

Crack criterion
To study the susceptibility of corner crack occurrence, ‘hoop stress’ $\sigma_h$ and ‘hoop strain’ $\epsilon_h$ components were calculated to show the transverse stress–strain component oriented parallel to the perimeter of the shell. To calculate these hoop values, first, the angle of the heat flux direction with respect to the global $x$ and $y$ axes is obtained from the temperature results. The stress–strain component perpendicular to that direction, i.e. $\theta = 90^\circ - \phi$, is then derived from

$$\sigma_h = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta . . . (11)$$

where $\sigma_x$ is the $x$ stress, $\sigma_y$ is the $y$ stress, and $\tau_{xy}$ is the shear stress.

Strand and mould domain
Figure 7 shows the finite element mesh of the 2D horizontal section of the billet strand and mould and its boundary conditions. A twofold symmetry assumption allows a quarter transverse section of the billet to be modelled. This domain consists of 5273 nodes and 5135 four node iso-parametric quadrilateral elements in the billet, and 207 nodes and 160 elements in the mould. The element equations are assembled using a single integration point, and the equations are solved using Newton–Raphson iteration. Further model details are given elsewhere.\textsuperscript{39} The boundary conditions used are also shown in Fig. 7. Further simulation conditions for the plant trial are described in Table 6.

Table 6 Simulation conditions for plant trial

<table>
<thead>
<tr>
<th>Steel grade</th>
<th>$C = 0.04\text{ wt-%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidus temperature</td>
<td>1529°C</td>
</tr>
<tr>
<td>Solidus temperature</td>
<td>1511°C</td>
</tr>
<tr>
<td>Superheat</td>
<td>25 K</td>
</tr>
<tr>
<td>Contact heat transfer coefficient</td>
<td>$2500 \text{ W m}^{-2} \text{ K}^{-1}$</td>
</tr>
<tr>
<td>Mould-water heat transfer coefficient</td>
<td>$29400 \text{ W m}^{-2} \text{ K}^{-1}$</td>
</tr>
<tr>
<td>Casting speed</td>
<td>$2.2 \text{ m min}^{-1}$</td>
</tr>
<tr>
<td>Taper</td>
<td>$0.75%$</td>
</tr>
</tbody>
</table>
Comparison of calculated stress profiles with analytical solutions

Comparison with plant trial

The 2D transverse slice model for simulating billet casting under the plane strain condition described above was validated by comparing with measurements from the plant trial, based on the conditions given in Table 6, featuring oil casting with a 4 mm corner radius.

Temperature

Axial mould-temperature profiles were calculated using both the AMEC2D and CON1D models. Figure 10 compares the predictions with the measured temperature profile down the mould, found by averaging the thermocouple values across each of the four rows. The heat flux profile in the CON1D model was adjusted carefully, to match the temperatures accurately. The AMEC2D model ignores axial heat conduction so is not expected to match exactly, but still agrees reasonably well. Figure 10 also includes the hot and cold face temperatures.

The internal consistency of the finite element model developed in the present work (AMEC2D) has been validated with analytical solutions under the condition of plane strain as shown in Fig. 7. Weiner and Boley\(^{41}\) developed an exact analytical solution of thermal stress during one-dimensional solidification of a semi-infinite elastic–perfectly plastic body after a sudden decrease in surface temperature. Table 7 gives the detailed conditions for verification of the analytical solution.

Figure 8 compares this solution with numerical calculations for various solidification times. Although the temperature profile of AMEC2D agrees closely with the analytical solution (Fig. 8a), the maximum tensile and compressive stresses are 6·5 MPa and −22·9 MPa, which differ from the analytical solution by 34% and 11.5%, respectively (Fig. 8b). This discrepancy is caused by the assumption of plane strain in AMEC2D, which is different from the true state of generalised plane strain in the analytical solution. However, comparing AMEC2D results with those of the CON2D model\(^{42,43}\) using a fine mesh size of 0·1 mm, as seen in Fig. 9, both show almost the same stress profile, which implies that the mesh size adopted in the present work is adequate.

Table 7 Simulation conditions for analytical solution test\(^{41}\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>7400 kg m(^{-3})</td>
</tr>
<tr>
<td>Specific heat</td>
<td>700 J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>33 W m(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>Latent heat</td>
<td>272 kJ kg(^{-1})</td>
</tr>
<tr>
<td>Initial temperature</td>
<td>1469°C</td>
</tr>
<tr>
<td>Liquidus temperature</td>
<td>1469°C</td>
</tr>
<tr>
<td>Solidus temperature</td>
<td>1468°C</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>1300°C</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>40 GPa</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0·35</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>20 x 10(^{-6}) K(^{-1})</td>
</tr>
<tr>
<td>Yield stress at surface temperature</td>
<td>20 MPa</td>
</tr>
</tbody>
</table>

Comparison with analytical solution

The corresponding heat flux profiles predicted by both models are compared in Fig. 11. The accurate CON1D model curve shows a slight dip and rebound in heat flux between ~20 and 100 mm below the meniscus. This is a result of the unexpected lower temperature measured by the highest thermocouple. It is interesting to note that this drop corresponds approximately to the region of negative mould distortion, suggesting that this negative taper at the meniscus might play a role. This heat flux dip phenomenon has been observed by others\(^{13,44,45}\). The AMEC2D curve shows the classic monotonically decreasing profile, which is more commonly observed.

The heat flux for the mould powder casting case is also included in Fig. 11. Its overall profile is much lower than that for the oil casting case. This result also agrees

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with other work. This lower heat flux is caused by the insulating effect of the mould flux layer between the mould and strand.

**Heat balance**

To validate the heat flux profiles, a comparison was made with an energy balance carried out for the cooling water. The model predictions of average heat flux, found from the areas under the curves in Fig. 11, are 1.84 and 1.80 MW m\(^{-2}\) for CON1D and AMEC2D, respectively. The measured cooling water temperature increase of 8 K corresponds to an average heat flux of 1.84 MW m\(^{-2}\), which agrees well with both model predictions.

**Solid shell thickness**

Figure 12 compares the measured solid shell thickness in a transverse section through the billet with the corresponding model prediction. The transverse section was taken at 285 mm below the meniscus, which corresponds to a simulation time of 7.8 s. The deformed shape of the strand is superimposed with temperature contours in Fig. 12. Shell thickness is defined in the model as the isotherm corresponding to the coherency temperature, assumed to be 70% solid. The general shapes of the predicted and measured solid shell match reasonably. It is noted that the model can also predict the re-entrant corner effect, observed in the sulphur print. This agreement appears to validate the remaining features of the present model, including air gap formation in the corner region.

The shell thickness is plotted in Fig. 13 as a function of residence time in the mould. Also plotted in Fig. 13 are the plant trial measurements, by means of the tracer test. It can be seen in Fig. 13 that the predicted solid shell growth is reasonable, considering the uncertainty about the penetration depth of the tracer into the mushy zone of the solidifying shell.
Bulging below mould

Bulging below the mould depends on the temperature and strength of the shell at the mould exit. In the mould, the surface temperature of the strand is governed by the contact between the strand and the mould, which defines the gap between them. This is influenced by the mould taper, so a simulation was also done for the extreme case of no mould taper. Figure 14 shows axial profiles of the surface temperature at the strand centre, corner, and 5 mm off corner. Regardless of taper, the centreline surface temperature has the same profile, decreasing monotonically to 900 °C at the mould exit (Fig. 14a). This is because the billet strand is always in good contact with the mould at the strand centre. The temperature rebound below the mould is simply due to the slower rate of heat removal by the sprays.

At the corner region, the temperature rebounds after ~1 s for both cases, owing to air gap formation. This time corresponds to initial formation of the air gap, and is delayed by applying the taper, as shown in Fig. 14c. An air gap still forms, because the taper of 0.75%/m is not sufficient to match the shrinkage of the shell. Figure 15 depicts transverse temperature profiles along the billet surface at various casting times, with taper. After the initial solidification stage (0.5 s), the temperature around the corner region is shown to remain higher throughout casting. This was not observed by Brimacombe et al., who did not simulate air gap formation during the calculation. They attributed off corner internal cracks to a hinging action around a cold, strong corner. However, Fig. 15 implies that the corner region has a higher surface temperature, which enhances hinging below the mould.

The strand shell exiting the mould is weak and hot, so the internal liquid pressure bulges the shell outwards below the mould. Although it might be supposed that this bulging in billet casting is small, compared with slab casting, the higher casting speeds and lack of support can make it significant. This bulging can cause internal strain in the shell, depending on billet geometry features such as corner radius and taper. Figure 16 shows the evolution of displacement at the centre and corner of the billet surface. As seen in Fig. 16, the bulging at the centre of the billet is predicted to be ~1.4 mm for 4 mm corner radius of billet with
Variation of shell profiles and temperature contours in corner region

(a) 4 mm corner radius; (b) 15 mm corner radius

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respect to displacement of the billet corner. Bulging of the billet during the plant trial was also measured, based on the distance from the billet centre to the non-bulged line extending between the two billet off corner locations (4 mm from each edge). These measurements were made on the cold section, and ranged from 0 to over 2 mm. Considering the uncertainties when evaluating the bulging, the calculated bulging amount seems to be consistent with the measured value.

**Stress and crack prediction**

To illustrate the stress state through the solidifying shell, transverse stresses $\sigma_t$ are plotted at various strand positions at 19 s of casting time (mould exit) in Fig. 17. The peak tensile stress is ~ 3 MPa, and is found beneath the surface. In the mould, it is similar around the billet perimeter except near the corner. The peak compressive stress is found at the surface, and is much higher at the centre region than at the off corner and corner. This is because of the huge drop of surface temperature, resulting from good contact between the strand and the mould, which increases the shell strength. The superimposed temperatures (Fig. 17) through the shell show that the peak stress clearly corresponds to the $\delta$ ferrite phase, as indicated by the horizontal lines. This agrees with the findings of Moitra et al., that the sudden shrinkage from the $\delta$ to the $\gamma$ phase produces these tensile peaks, which may cause subsurface cracks.

During the plant trial, billet samples were also taken under the same casting conditions as described in Table 1, and their microstructure was investigated. Figure 18 compares the typical microstructure of an off corner billet that was found in this plant trial with stress and strain development at 100 mm below the mould exit. Usually, solidification cracking or hot tearing can occur when the steel in the mushy zone is under tension beyond some small critical limit, owing to the existence of a liquid film. Peak hoop tensile stresses, which pull apart dendrites and result in hot tears, take place both at the centre and at the off corner of the billet, as shown in Fig. 18b. Effective plastic strain is highest at the off corner location (Fig. 18c). It is interesting to note that the peak strain occurs in a region of tensile hoop stress, and corresponds roughly to the position of crack occurrence. The exact location of this crack obviously matches the surface depression.

**EFFECT OF MOULD CORNER RADIUS**

Next, the model was applied to compare the thermomechanical behaviours of steel cast in 4 mm (small) and 15 mm (large) corner radius moulds. The results have been evaluated according to the effects on heat transfer and gap formation, longitudinal corner surface cracks, and longitudinal off corner subsurface cracks.

**Heat transfer**

Figure 19 shows temperature contours with the deformed shapes of both billets near the corner region, at four locations down the mould. Both billets experience increasing solid shell thinning at the corner, and the associated evolution of an air gap, with increasing casting time. During initial solidification, a uniform solidifying shell forms as a result of the contact between the strand and mould. After less than 1 s, the shell starts to shrink away from the billet and an air gap forms near the corner. This reduces the local heat flow from the strand to the mould. This raises the temperature of the corner regions 22 mm below the meniscus, as shown in Fig. 14. Closer examination of the temperature profile around the corner reveals that the 15 mm corner radius billet develops both higher surface temperature at the corner and more severe non-uniform temperature contours along the billet surface as solidification proceeds. This re-entrant corner effect persists even below the mould exit.

**Longitudinal corner surface cracks**

Figure 21 compares contours of hoop stress and hoop plastic strain of both billets near the corner region at the casting time of 8 s. As can be seen in Fig. 21, both hoop values are much higher in the 15 mm radius billet. The development of hoop plastic strain with time is shown in Fig. 22 at a critical corner location, 1 mm beneath the corner surface, where longitudinal corner cracks were found. Figure 22 reveals that the large corner radius billet develops tensile plastic strain from 4 to 14 s in the mould (150–520 mm below the meniscus). This is consistent with breakout shell observations, in which corner cracks begin some distance below the meniscus. Compression is found both before and after this time. Below the mould, bulging causes the shell to hinge around the corner, forcing the corner surface into compression. The small radius billet experiences compressive plastic strain at this location throughout casting, owing to two-dimensional cooling at
EFFECT OF CASTING WITH MOULD FLUX

Finally, a simulation was carried out to study the effect of mould powder lubrication on the thermomechanical behaviour of steel cast in the two different corner radius moulds but with the same inadequate linear taper. Figure 25 compares the solid shell contours at the mould exit. Both billets show more uniform shell solidification with mould flux, leading to a smaller air gap size, despite having a thinner average shell owing to the lower heat flux associated with a thicker gap. The smaller air gap size is a result of less shrinkage of the hotter shell. In oil casting, this extra uniformity could be achieved by increasing the taper.

Changing the lubricant from oil to powder does not change the nature of the stress and strain development, or the susceptibility of large and small corner radius billets to corner and off corner cracks, respectively. The 15 mm corner radius billet develops peak hoop stress and strain at the corner and the 4 mm corner radius billet generates both peaks at the off corner region. Figure 26 shows the evolution of hoop stress and strain with casting time for the 15 mm corner radius billet. With powder, the heat flux is lower and the solidifying shell is hotter and weaker. Thus, all of the stresses and strains, and the associated surface defects, are exacerbated slightly.
23 Hoop stress and hoop plastic strain contours for 4 mm corner radius: oil casting

24 Hoop stress and hoop plastic strain contours for 15 mm corner radius: oil casting
In the present analysis, the flux layer is assumed to maintain constant thickness during gap formation. In reality, it is likely that liquid flux will build up to fill the gap. This would increase the corner heat flux relative to the predictions here, which would give rise to even more uniform shell thickness. Therefore, for the same average heat flux and shell thickness at the mould exit, the powder casting practice is expected to be less susceptible to cracks, owing to better uniformity of the solidifying shell.

**MECHANISM OF LONGITUDINAL CRACK FORMATION**

The numerical analysis carried out in the present study indicates two distinct mechanisms to generate longitudinal corner cracks or longitudinal off corner internal cracks in the casting of steel billets with inadequate linear taper. Longitudinal corner cracks are predicted to arise only in large corner radius billets, owing to tension developing across the hotter and thinner shell along the exact centre of the corner during solidification in the mould. Such surface cracks could extend deeper because of solid shell bulging both in the mould, owing to mould wear, or below, owing to poor alignment of the guide rolls. On the other hand, small corner radius billets allow the formation of a thinner shell at the off corner region inside the mould. This exacerbates the hinging action that accompanies bulging below the mould. This causes high plastic tensile strain across the dendrites in the off corner region, leading to longitudinal subsurface off corner cracks in billets cast in these moulds.

Although the above analysis ignores the important effects of asymmetry, rhomboidity phenomena, and lower ductility from copper pickup on these defects, these mechanisms suggest more about mould operation. Applying mould powder as the lubricant allows the shell to solidify more uniformly, which could potentially reduce both of these types of cracks. Employing an optimised parabolic mould taper could achieve the same benefit. Mould wear effectively reduces the taper and probably worsens both cracking problems. Mould wear at the corner would cause a more severe gap, leading to a hotter and thinner shell there, which would increase susceptibility to corner surface cracks. Mould wear at the centre would allow billet bulging to occur inside the mould. This could allow the hinge action inside the mould, and increase susceptibility to off corner subsurface cracks. Mould wear at the centre would allow billet bulging to occur inside the mould. This could allow the hinge action inside the mould, and increase susceptibility to off corner subsurface cracks. Mould wear at the centre would allow billet bulging to occur inside the mould. This could allow the hinge action inside the mould, and increase susceptibility to off corner subsurface cracks.

Finally, the present work suggests that mould corner radius controls how longitudinal cracks are manifested, but is not the root cause of the problem. This means that large corner radius moulds could be used effectively to improve smooth rolling operations while still maintaining quality billets free of longitudinal cracks, as long as other casting parameters are optimised. Specifically, an optimised parabolic mould taper should be employed together with a well maintained mould shape (free of wear and permanent distortion),
a hoop stress; b hoop plastic strain

26 Evolution of calculated stress and strain contours for 15 mm corner radius with powder casting

mould powder lubrication, and adequately aligned foot rolls. More study is needed to achieve these requirements for different casting speeds, section sizes, and mould lengths.

CONCLUSION
Using a two-dimensional coupled thermoelastoviscoplastic finite element model of a slice through the continuous cast strand, the thermomechanical behaviour of a square billet has been analysed. Calculated results of temperature of the mould, heat flux, thickness of the solidifying shell, bulging deformation, and location of longitudinal crack formation are in good agreement with experimental observations. The following conclusions are based on simulations of 4 mm and 15 mm radius corners of 120 mm square billets of low carbon steel with only 0.75% Cr/m linear taper and cast at 2.2 m min⁻¹:
1. A gap forms in the corner region of linear taper moulds owing to insufficient taper.
2. As the corner radius of the billet increases from 4 to 15 mm, this gap spreads further around the corner towards the centre of the strand and becomes larger. The accompanying drop in heat flux leads to more non-uniformity in temperature around the billet perimeter as solidification proceeds.
3. Longitudinal corner cracks are predicted only in the large corner radius billet. They form as a result of tension within the hotter and thinner shell along the corner during solidification in the mould (150–520 mm down the mould). These surface cracks could extend deeper by solid shell bulging owing to mould wear, or poor alignment of guide rolls below the mould.
4. Longitudinal off corner subsurface cracks are predicted to form more easily in the small corner radius billet. They are caused by hinging of the thin, weak shell around the corner at the off corner region, as a result of bulging allowed either in the mould by mould wear, or below the mould by poor guide roll alignment.
5. Changing from oil lubrication to powder casting with good infiltration and high gap conductivity and/or optimising mould taper leads to a more uniform shell in the mould, with potential benefits for reducing longitudinal cracks.
6. With optimised parabolic taper, no mould wear, proper powder lubrication, and adequate submould guide roll support, large corner radius billets should be castable without longitudinal cracks, with the benefit of a smoother corner for rolling operations.

**APPENDIX**

The gap size of 4 mm and 15 mm corner radius moulds for casting 120 mm square billets was approximated geometrically assuming:

(i) shrinkage is 0·5%·
(ii) circumferential length of the gap is 11·78 mm

Figure 27a shows a schematic diagram of the corner region of a 15 mm radius billet, where $BF$ is the initial radius (15 mm), $AF = AD$ is the new radius, $EF$ is the initial half perimeter of the 15 mm radius, and $DF$ is the new half perimeter of the new radius.

From Fig. 27a, the following relationships can be obtained

$$\frac{\theta}{\sin \theta} = \frac{EF}{BF} \sin 45$$  \hspace{1cm} (12)

$$\frac{\pi}{2} = \frac{\theta}{\sin \theta} = \frac{BF}{EF} \sin 45$$  \hspace{1cm} (13)

$$\frac{\pi}{2} = \frac{\theta}{\sin \theta} = \frac{BF}{EF} \cos 45$$  \hspace{1cm} (14)

From equations (12)–(14), the gap size for a 15 mm radius billet, $DE = BF + 15 - AD$, is found to be 1·3 mm.

Figure 27b shows the corner region of a 4 mm radius billet. Assuming the same circumferential gap length, the billet perimeter can be divided into three parts, a straight part $FG$, an angled part $FI$, and the 4 mm radius part as indicated.

The new half perimeter $FG + FI + ID$ is $0·5\%$ less than the initial half perimeter $GHE$, expressed by

$$FG + (FI + ID)^{1/2} + \frac{\pi}{4} (BE - DE) = FHE(1 - 0·5\%)$$  \hspace{1cm} (15)

Solving equation (15), the gap size $DE = HI$ for a 4 mm radius billet is 0·79 mm.

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