Evaluation of Turbulence Models in MHD Channel and Square Duct Flows

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Acknowledgements

• Continuous Casting Consortium Members (ABB, Arcelor-Mittal, Baosteel, Corus, LWB Refractories, Nucor Steel, Nippon Steel, Postech, Posco, ANSYS-Fluent)

• Aaron Shinn, Lance Hibbeler and other graduate students at Metals Processing Simulation Lab.
Importance of MHD and non-MHD turbulent flows in continuous casting of steel

- Continuous casting is an energy intensive process and needs careful process optimization to avoid defects.
- Flow is inherently turbulent due to wobbling jets.
- Turbulence is critical to the defects.
  - Too less turbulence leads to meniscus freezing and thus hook formation.
  - Too high turbulence leads to slag entrainment and alumina entrapment in the shell.
- Complex physics due to multiphase flows (argon gas and inclusions) further complicates the issues.
- Nozzle geometry and magnetic fields, if used wisely, can control turbulent flow and minimize defects in the product.

Investigation methods for turbulent flows

- In safe accessible processes:
  - Experiments are possible but usually difficult to conduct, even at relatively few locations.
    - High cost
    - High labor
- In unsafe and harsh processes: such as continuous casting of steel operating at 1500°C,
  - Experiments are too difficult and sometimes impossible.
- Options available:
  - A replica model for experiments at the same or lower scale with a fluid easy to handle
  - Computer simulation, can give large data almost inexpensively.
Computer simulations of turbulent flows

- Various methods are available to model turbulent flows
  - Direct Numerical Simulation (DNS)/Large Eddy Simulation (LES)
  - Reynolds-Averaged Navier-Stokes (RANS)
- DNS/LES: not always possible
  - The large range of scales (very small (nozzle bore) to very large (mold width) dimensions) requires fine mesh leading to high computational and storage costs.
  - The complex multiphase physics involved with these processes, DNS/LES of multiphase flows need extensive efforts and sometimes impossible.
- RANS models are cheap and the most viable option to model turbulence involving complex multiphase flows in these systems.

RANS models for MHD and non-MHD turbulence

- After Reynolds averaging in RANS models, the Navier-Stokes (N-S) equations need closure of Reynolds stresses to model turbulence.
- Closures of Reynolds stresses:
  - Mostly mathematical and are developed based upon physical understanding of the turbulence.
  - The various constants of in these closures are tuned through experiments in simple geometries.
  - Need extensive testing in pertinent geometries for the predictions of turbulence, mean velocities, frictional losses etc.
- The effect of magnetic field on turbulence in these closures is not well incorporated.
  - Kenjereš et al proposed generalized formulation for MHD effects, but only tested extensively in channel flow that too on tailored low-Re k-ε and RSM models
  - Need proper testing for turbulence in other models as well as in two-wall bounded turbulent flows.
Objective

- Test turbulence models of k-$\varepsilon$ and RSM category, as available in FLUENT, for MHD and non-MHD turbulent flows
  - Low-Re models (Abid, LB, LS, YS, AKN, CHC, RSM-stress-omega)
  - High-Re models (RKE, RNG, SKE, RSM-linear-pr-strain)

- Test various wall treatment methods as available in FLUENT
  - Standard wall functions (SWF) ($30<y+<500$)
  - Non-equilibrium wall functions (NEWF) ($30<y+<500$)
  - Enhanced wall treatment (EWT) (no limitation on $y+$)

- Test flow geometries with DNS databases: Wall attached flows: MHD and non-MHD flows
  - Channel
  - Square duct

Relevance to the Continuous Casting of Steel

- Continuous casting process consist of the turbulent flows with the following features
  - High speed flow separation behind slide gate, stopper-rod and at the top of the ports.
  - Wall attached flow in the SEN bore.
  - Flow in a rectangular cross-section in the mold.
  - Slanted jet impinging at the narrow faces in the mold.

- Requires testing of various models in the following pertinent geometries
  - Channel flow/pipe flow
  - Flow in a square or rectangular cross-sections
  - Back-ward facing step
  - Impinging jet/slanted impinging jets

- In the current work, focus has been kept on testing various k-$\varepsilon$ and RSM models for hydrodynamic and MHD turbulence in
  - Channel
  - Square duct
Types of RANS models

- RANS models are broadly classified into two categories
  - Models using Boussinesq hypothesis to model Reynolds stresses in terms of mean velocity gradients and eddy viscosity. (mainly two-equation models, Realizable(RKE), RNG, standard(SKE) k-ε, k-w and low-Re k-ε models)
    - Assumptions: Isotropy of Reynolds normal stresses and eddy viscosity
  - Directly solving for transport equations for six independent Reynolds stresses. (seven-equation Reynolds Stress Models(RSM))
    - Can handle anisotropy of Reynolds normal stresses, but needs solution of more equations.

Near-wall behavior in wall-bounded turbulent flows

- In addition to the turbulence models, the near-wall treatment is very important in wall bounded turbulent flows.
  - Wall boundary (on macro scales):
    - Always has no-slip boundary condition
    - The viscous damping and kinematic blocking of turbulence leads to high velocity gradient.
    - High velocity gradient close to the wall is the main source of turbulence production.
Near-wall features in wall bounded turbulent flows

- Near-wall has three regions:
  - Viscous sublayer (molecular viscosity is imp)
    - Universal behavior, $U^+ = Y^+$, constant shear stress, $y^+ < 5$.
  - Buffer layer (both molecular and turbulence are imp, they overlap) (blend of the two) $5 < Y^+ < 30$.
    - For $y^+ < ~11$, $U^+ = Y^+$ is more accurate, and $y^+ > ~11$, log-law.
  - Turbulent log-law layer (fully turbulent region)
    - Universal behavior, $U^+ = (1/k)\ln(Y^+)+B$, $30 < Y^+ > 500$.

\[ y^+ = \frac{y u_t}{V} \]  
\[ U^+ = U/\bar{u}_t \]

- Turbulent log-law layer (fully turbulent region)
  - Universal behavior, $U^+ = (1/k)\ln(Y^+)+B$, $30 < Y^+ > 500$.

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\[ k \text{ and } B \text{ are different for smooth and rough walls.} \]

- Low-Re models do not need any special wall treatment, rather damping functions are used within the model equations and eddy viscosity formulations to handle near-wall behavior.

- Require to resolve typically up to the viscous layer in the near-wall region, i.e. $y^+$ at the cell center next to the wall should be $\leq 1$. 

\[ y^+ = \frac{y u_t}{V} \]
\[ \tau_w = \rho u_t^2 \]
Near-wall treatment in high-Re models (RKE, RNG, SKE, RSM-linear-pr-strain)

- Standard wall functions (SWF)
  - Requires $30<y^+<500$, applies in fully turbulent region.
- Non-equilibrium wall functions (NEWF)
  - Standard wall function sensitized to pressure variations
  - Again requires $30<y^+<500$, applies in fully turbulent region.
- FLUENT uses $U^*$ and $y^*$ (in SWF and NEWF) which are approximately equal to $U+$ and $Y+$ in equilibrium (generation and dissipation of $k$ are equal) boundary layers respectively.
- FLUENT applies log-law in SWF and NEWF when $y^*>11$.
- Enhanced wall treatment (EWT)
  - Uses two-layer modeling
    - $U^*$ and $y^*$ are defined based upon kinematic viscosity, friction and turbulent velocity.
    - Below, $Re_{turbulent}<200$, 1-d model of Wolfstein, otherwise default turbulence model
    - Uses a linear blended laminar ($U+=Y+$) and turbulent ($U+=(1/k)\ln(y+)+B$) behavior

MHD sources to turbulence (Kenjereš et al, phy fluids (2004) & IJHFF(200))

MHD sources for $k$-$\varepsilon$ models

$$S_k^M = -\sigma \tilde{B}_0^2 k \exp\left(-C_1^M \frac{\sigma \tilde{B}_0^2 k}{\varepsilon}\right)$$

$$S_{\varepsilon}^M = -\sigma B_0^2 \varepsilon \exp\left(-C_1^M \frac{\sigma \tilde{B}_0^2 k}{\varepsilon}\right)$$

$$C_1^M = 0.025$$

MHD sources for RSM model (can be calculated by deriving Reynolds stress budget equations)

$$S_{w'w'}^M = \sigma \left(-2B_{y0}^2 w' \frac{\partial \phi'}{\partial x} - 2B_{y0}^2 w' w' \right)$$

$$S_{v'v'}^M = 0$$

$$S_{w'v'}^M = \sigma \left(2B_{y0} w' \frac{\partial \phi'}{\partial x} + B_0^2 w' v' \right)$$

$$S_{v'w'}^M = \sigma \left(-B_{y0} w' \frac{\partial \phi'}{\partial x} - B_0^2 w' v' \right)$$

$$S_{v'v'}^M = \sigma \left(-B_{y0} v' \frac{\partial \phi'}{\partial x} - B_0^2 w' v' \right)$$

$$S_{w'v'}^M = \sigma \left(-B_{y0} v' \frac{\partial \phi'}{\partial x} - B_0^2 w' v' \right)$$

$$\frac{\partial \phi'}{\partial x} = \beta \varepsilon_{kmm} u' u'_m B_{x0} \Rightarrow u'_i \frac{\partial \phi'}{\partial x} = \beta \varepsilon_{kmm} u'_i u'_m B_{x0}$$

$$\varepsilon_{kmm} = \begin{cases} 1 & \text{if (k, m, n) are cyclic} \\ -1 & \text{if (k, m, n) are anticyclic} \\ 0 & \text{otherwise} \end{cases}$$

$$\beta = 0.6$$

Implemented using User Defined Functions (UDF) in FLUENT.
**DNS database: High-Re non-MHD channel flow**

Various parameters in high Reynolds number non-MHD channel DNS calculation

<table>
<thead>
<tr>
<th>Re ($=DW_b/\nu$)</th>
<th>Grid ($N_x \times N_y \times N_z$)</th>
<th>Comput. domain</th>
<th>Spatial resolution ($\Delta x$, $\Delta y$, $\Delta z$)</th>
<th>Mag. field orientation</th>
<th>Ha</th>
<th>$W_b/\Delta p/dz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>45818 (Re=1120)*</td>
<td>1024x1024x768</td>
<td>$\pi x1x2.5\pi$</td>
<td>9.16, 0.163-4.25, 17.2</td>
<td>-</td>
<td>0</td>
<td>20.45 / 2.0</td>
</tr>
</tbody>
</table>

* where, $Re_\tau = 0.5 Du_\tau/\nu = \delta u_\tau/\nu$

$\delta=0.5$, is half channel height.

$Ha = B_y 0.5D \sqrt{\frac{\sigma}{\rho \nu}}$

S.-I. Satake, T. Kunugi, K. Takase, and Y. Ose, Direct numerical simulation of turbulent channel flow under a uniform magnetic field for large-scale structures at high Reynolds number, Phys. Fluids, 2006, 18, 125106.

**DNS database: Low-Re MHD and non-MHD channel flows**

Various parameters in low Reynolds number MHD and non-MHD channel DNS calculations

<table>
<thead>
<tr>
<th>Re ($=DW_b/\nu$)</th>
<th>Grid ($N_x \times N_y \times N_z$)</th>
<th>Comput. domain</th>
<th>Spatial resolution ($\Delta x$, $\Delta y$, $\Delta z$)</th>
<th>Mag. field orientation</th>
<th>Ha</th>
<th>$W_b/\Delta p/dz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4586 (Re=150)**</td>
<td>128x97x128</td>
<td>$\pi x1x2.5\pi$</td>
<td>7.36, 0.08-4.91, 18.4</td>
<td>-</td>
<td>0</td>
<td>15.28 / 2.0</td>
</tr>
<tr>
<td>(Iwamoto et al)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4710 (Re=150)**</td>
<td>64x128x64</td>
<td>0.5$\pi x1x1.25\pi$</td>
<td>7.36, 0.08-4.9, 9.2</td>
<td>$B_y$</td>
<td>6.0</td>
<td>15.7 / 2.0</td>
</tr>
<tr>
<td>(Noguchi et al)</td>
<td></td>
<td></td>
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</tbody>
</table>

** where, $Re_\tau = 0.5 Du_\tau/\nu = \delta u_\tau/\nu$

$\delta=0.5$, is half channel height.

$Ha = B_y 0.5D \sqrt{\frac{\sigma}{\rho \nu}}$


http://www.thtlab.t.u-tokyo.ac.jp/
DNS database: Low-Re MHD and non-MHD square duct flows

Various parameters in low Reynolds number MHD and non-MHD square duct DNS calculations

<table>
<thead>
<tr>
<th>Re ((=DW_b/ν))</th>
<th>Grid ((N_xN_yN_z))</th>
<th>Comput. domain</th>
<th>Spatial resolution ((Δx', Δy', Δz'))</th>
<th>Mag. field orientation</th>
<th>Ha</th>
<th>(\frac{W_b}{d\rho/ dz})</th>
</tr>
</thead>
<tbody>
<tr>
<td>5466 ((Re_τ=360)***) ((Shinn et al))</td>
<td>160x160x1024</td>
<td>1x1x8</td>
<td>1.47-3.24, 1.47-3.24, 2.81 (1%) stretching in x- and y-</td>
<td>-</td>
<td>0</td>
<td>15.187 / 4.0</td>
</tr>
<tr>
<td>5602 ((Re_τ=361)***) ((Chaudhary et al))</td>
<td>128x128x512</td>
<td>1x1x16</td>
<td>1.41-4.92, 1.41-4.92, 11.28 (2%) stretching in x- and y-</td>
<td>(B_y)</td>
<td>21.2</td>
<td>1.057/0.01857</td>
</tr>
</tbody>
</table>

*** where, \(Re_τ = Du/ν\)  
D=1, is the side of the square duct.  
\[ Ha = B_y D \left( \frac{\sigma}{\rho v} \right) \]


Computational domains, boundary conditions and process parameters in DNS and RANS calculations for channel and square duct flows

DNS: Domain dimensions are given in the tables.  
RANS: Domain size of 1x1x1 for both channel and square duct.

1. Same process parameters as given in the tables for DNS are used in RANS calculations.
2. The flow rate/bulk Reynolds number was fixed corresponding to the DNS and pressure gradient was allowed to change.
3. Channel: Streamwise (z-) and spanwise(x-) directions are considered periodic.
4. Square duct: Streamwise (z-) direction is considered periodic.
5. Walls are taken no-slip and electrically insulated.
6. FLUENT’s segregated solver with SIMPLE method for pressure-velocity coupling.
7. 1st and 2nd order upwind schemes for convection.
8. Convergence was pursued until un-scaled residuals became stagnant and reached below \(~1x10^{-03}\).
RANS models and wall treatment for various flows

- **High-Re flow (Re~50,000):** Non-MHD Channel
  - Models: RKE, SKE, RNG, and RSM-linear-pressure strain models
  - Wall treatment: Standard wall function, Non-equilibrium wall function, enhanced wall treatment

- **Low-Re flow (Re~5000):** MHD and non-MHD Channel and Square duct
  - Wall treatment: Enhanced wall treatment

- **Standard and non-equilibrium wall functions were not used for low-Re flows,**
  - Re$_t$=150 corresponds to Re=4586, the number of cells required to have y$^*$>30 (in the cells next to wall) are ~5, which are too less to use, therefore standard and non-equilibrium wall function approaches are not appropriate for such low-Re flows.
  - Either enhanced wall treatment or low-Re models are only appropriate.

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**High-Re (Re=45818):** Non-MHD Channel flow: Grid independence and mesh selection

1. **5 grids:** 3 uniform and 2 non-uniform
   - 50x10x10(uniform), 80x10x10(uniform), 130x10x10(uniform), 139x10x10(non-uniform), 208x10x10(non-uniform)

2. **TKE obtained grid independence as y+ approached ~1** (i.e. 139x10x10).

3. **Results changed significantly close to the wall from y+=22 to 9,** perhaps due to two layer modeling and single blended wall law.

4. **Other models (i.e. RNG, SKE, RSM-linear-pr-strain) also obtained grid independence at the same mesh.**
1. **3 grids**: 2 uniform and 1 non-uniform
   - 50x10x10 (uniform), 80x10x10 (uniform), 100x10x10 (non-uniform)

2. As mesh is refined from 50x10x10 to 80x10x10, almost all models obtained grid independence in the major part of profile.

3. The slight variations are seen close to the wall upon further refinement from 80x10x10 to 100 (non-uniform) x10x10.

4. This time EWT behaved smoothly due to small range of $y^+$ (i.e. from 3.2 to 0.9).

5. Other models (i.e. RKE, SKE, RNG-with diff viscosity, RSM-linear pressure strain) also obtained grid independence at the same mesh.
Meshes in final non-MHD channel simulations

• High-Re(45818):
  – With standard and non-equilibrium wall functions since \( y^+ \) has to be kept >30 therefore a uniform mesh of size 30x10x10 which keeps \( y^+ \)~36-38 is used.
  – Enhanced wall treatment: grid independent mesh: 139x10x10 (\( y^+ \)~1).

• Low-Re(4586):
  – Enhanced wall treatment: grid independent mesh: 100x10x10 (\( y^+ \)~1).
  – Low-Re models: grid independent mesh: 120x10x10 (\( y^+ \)<1).

Models and Meshes for MHD channel and non-MHD and MHD square ducts

• Models:
  – MHD(4710) channel, non-MHD(5466) and MHD(5602) square ducts: all low-Re
  – Therefore same models as in low-Re non-MHD channel

• Mesh:
  – MHD channel: Same grid independent mesh as in non-MHD channel
  – MHD and non-MHD square duct: same wall normal grid independent mesh, as in low-Re non-MHD channel(4586), is taken in the horizontal as well as in the vertical directions.
High-Re(45818) Non-MHD channel: TKE

1. TKE matched closely with the DNS in the core but errors increased from the core towards the wall.

2. The peak is underestimated by 22-27% in EWT and ~42% in SWF/NEWF.

3. Performance of RKE is not as good as of others.

High-Re(45818) Non-MHD channel: \( u_{\text{rms}}, v_{\text{rms}}, w_{\text{rms}} \)

1. SWF and NEWF performed similar, perhaps due to wall attached flow.

2. Again as in TKE, the errors in predictions increased from the core towards the walls.

3. It is interesting to note that the velocity fluctuations in the horizontal direction (i.e. x-, spanwise, wall free) matched much better with the DNS.

4. EWT performed better than SWF and NEWF.
High-Re(45818) Non-MHD channel: mean axial velocity and pressure losses

1. Mean axial velocity is predicted equally well by RSM and SKE with EWT, predictions with SWF are not as good as with EWT.

2. EWT resolves all the way up to viscous sublayer \((y^+ = 1)\), SWF has \(y^+ \approx 36-37\).

3. Similar predictions are given by models with SWF and NEWF therefore only with SWF is plotted.

4. All the models with EWT gives superior predictions of pressure gradient than SWF/NEWF.

Low-Re(4586) non-MHD channel: low-Re models: TKE and mean axial velocity

1. All low-Re models performed similar except LS, CHC and YS.

2. TKE was matched closely at the peak value but overpredicted in the core.

3. LS overpredicted across the whole length.

4. Although, CHC matched in the core but too much underpredicted peak value.

5. Overall, LB seems to be performing better but the right trend is given by YS.

6. Better performing models from TKE comparisons are selected to compare mean velocity.

7. All performed equally well matching mean axial velocity closely with the DNS.
Low-Re(4586) non-MHD channel: TKE and RMS of velocity fluctuations

1. All models gave similar performance in the prediction of TKE.
2. TKE is closely matched at the peak but overpredicted (almost 100%) in the core.
3. RSM-stress-omega (low-Re RSM model) gives superior prediction of TKE but does not capture the anisotropy of Reynolds stresses properly.
4. Right trend of the Reynolds stresses is captured by RSM-linear-pressure strain model.

Low-Re(4586) non-MHD channel: mean axial velocity and pressure gradient

1. Low-Re model (LB) gives superior prediction of pressure gradient.
2. Low-Re, LS model is the worst.
3. RKE, RNG, SKE with EWT predicted similar pressure gradient.

1. All(RKE, SKE, RSM) matched mean velocity closely.

All the models predicted pressure gradient within 20% error except LS overpredicting by 95%.
1. The models (LB, SKE/RKE, RSM-linear-pr-strain) which performed better in non-MHD channel are used in MHD channel, non-MHD square duct and MHD square duct.

2. LB with MHD sources matched the TKE exactly in the core but underpredicted the peak value.

3. Peak value is better predicted by LB without sources and SKE, RSM models with and without sources.

4. Effect of MHD sources is clear in suppression of TKE.

5. Weak effect of MHD sources is seen in the models using wall treatment (i.e. EWT).

1. The LB model with MHD sources matched mean velocity closely with the DNS, except missing some part in buffer and log region.

2. This behavior is consistent in TKE and mean velocity in this range of $y^+$.

3. The second best prediction is by the same LB model but without MHD sources.

4. The performance of RSM and SKE is similar with RSM performing slightly better.

5. The underprediction of normalized velocity by various models in the core is mainly due to the higher frictional losses leading to higher friction velocity in these models and thus causing the difference.

6. The SKE and RSM models do not show much effect of MHD sources in the normalized mean velocity.
Low-Re(Re=4710, Ha=6) MHD channel: MHD sources to TKE and $\varepsilon$

1. LB low-Re $k$-$\varepsilon$ model matches the source in $k$-equation better, especially in the core, followed by RSM and then SKE.

2. Although, SKE predicts the peak closely but overpredicts in the core.

3. Interestingly, all the models missed the source (i.e. +ve value) in $S^M_{\varepsilon, k}$ due to magnetic field very close to the wall (within $y^*<5$).

4. LB gives best qualitative prediction of MHD source to $\varepsilon$ equation.

5. SKE and RSM gives better values for $\varepsilon$ source but peak is not predicted at the wall.

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Low-Re(Re=4710, Ha=6) MHD channel: Pressure gradient

1. LB model with MHD sources predicted the best pressure gradient followed by same model without sources.

2. Performance by other models (RKE, SKE, RSM) with and without MHD sources is similar.

3. Same is confirmed on the previous slides.
Low-Re(Re=4710, Ha=6) MHD channel: MHD sources to ww and uu transport equations

1. In $S_{\text{M}^+\text{ww}}$, RSM behaves similar to as in turbulent kinetic energy source.

2. It underpredicts the peak value and overpredicts in the core.

3. The positive values of the source in $S_{\text{M}^+\text{ww}}$ below $y^+<5$ are again missed by the model.

4. The MHD source in $S_{\text{M}^+\text{uu}}$ is qualitatively captured but the values are overpredicted across the whole length.

Low-Re(5466): non-MHD square duct: TKE and RMS of velocity fluctuations

1. LB performed better followed by SKE, RKE and then RSM.

2. RSM-linear pressure-strain model with enhanced wall treatment overpredicts all the components of Reynolds normal stresses in the core.

3. Match is better at the peak values in both TKE and RMS of velocity fluctuations.
Low-Re(5466): non-MHD square duct: mean axial velocity

1. RKE, SKE and LB all performed similar.

2. RSM showed minor underpredictions in the core while matching with DNS close to the wall along bisector.

3. None of the models is able to predict the hat shape of the DNS along the diagonal.

3. RSM predicted closely in and around the corners, perhaps due to being able to resolve anisotropy of Reynolds stresses and secondary flows but underpredicted in the core.

4. All the k-ε models, like LB, RKE, and SKE, gives hemispherical profile and misses the shape of the profile around the corners as well as at the center.
Discussion on the mean axial and secondary velocities in non-MHD square duct (Re=5466)

- All the predictions, except Gavrilakis (which has Re=4410), are at the same bulk Reynolds number (Re=5466).
- Although, the two DNS estimates (Shinn et al and Gavrilakis) have different Reynolds number but they match qualitatively well for the mean axial velocity as well as for the secondary velocities.
- As can be seen, the secondary flows and their effects on the axial velocity are completely missed by both k-ε models (i.e. LB and RKE).
- The RSM predicted secondary velocities closely but does not show their effects quite well on the axial velocity and misses the bulging of axial velocity in the wall bisector regions.
- The predictions of axial velocity in both k-ε models (RKE and LB) are quite similar.

Low-Re(5466): non-MHD square duct: friction factor

1. The effect of secondary flows is clear on altering the axial velocity leading to friction factor profile with three peaks (two at the sides and one at the center).
2. The LB and RKE models are able to match the friction factor up to 0.1 unit distance from both side walls but are unable to predict the sagging regions, caused by returning secondary flows, on both sides of the center peak.
3. Both k-ε models overpredict in the core with only one center peak.
4. RSM model although qualitatively imitates the side peaks but does not give any peak at the center.
5. The side peaks in RSM, which are caused by secondary flows, are too much overpredicted.
1. In consistent with the friction factor, the RSM gives highest pressure gradient among all the models.

2. SKE and RKE give almost same pressure losses.

3. The LB model matches better than others with the DNS.

4. All the models predicted within 25% of DNS predictions.

5. LB is the closest with 12.5% higher estimates than DNS.
Discussion on TKE predictions in MHD square duct (Re=5602, Ha=21.2)

• TKE is suppressed more strongly close to the top wall than side wall and only LB model with MHD sources is able to predict this trend.

• LB without MHD sources does not perform that well and estimates minor differential suppression of turbulence.

• The MHD sources significantly improved the predictions, especially with LB low-Re k-ε model.

• RKE and RSM models overpredict TKE in the core along both the bisectors and do not show strong differential suppression.

• Both Mag-ind and Elect-pot give same predictions.

• Match in TKE is better close to side walls.

Low-Re(Re=5602, Ha=21.2): MHD square duct: RMS of velocity fluctuations

1. RSM model is able to capture qualitative trends of Reynolds stresses but overpredicts the values.

2. The closest match is achieved along horizontal bisector close to side walls where the effect of Lorentz force is the weakest.
Low-Re(Re=5602, Ha=21.2): MHD square duct: mean axial velocity

1. All the models predicted too much velocity flattening along vertical bisector.

2. Although LB with MHD sources matched TKE better but for mean velocity did not perform well, especially along vertical bisector.

3. Similar to TKE match is better in the axial velocity close to side walls.

Low-Re(Re=5602, Ha=21.2): MHD square duct: mean axial velocity along diagonal

1. The DNS shows, similar to non-MHD duct, a hat shape profile but this time the central dome is little suppressed and round.

2. RSM predicts mean velocity better.

3. LB and RKE missed the side humps.
Low-Re(Re=5602, Ha=21.2): MHD square duct: mean axial velocity contours with secondary velocity vectors

1. Secondary flow and mean axial velocity contours are significantly altered with the magnetic field.

2. None of the model is able to capture the right trend (especially strong bulging close to top and bottom walls).

3. RSM captures almost symmetric secondary flows.

4. LB and RKE does not predict any secondary flows.

5. LB and RKE overpredict the velocity flattening in the vertical direction.

Low-Re(5602, Ha=21.2): MHD square duct: MHD sources to TKE equation

1. Velocity-electric potential gradient correlation acts as the source whereas Reynolds normal stresses perpendicular to the magnetic field as sinks.

2. Source and sink follow similar profiles but the sink is stronger than source thus net effect being the suppression of the turbulence.

3. The TKE source due to MHD along both bisectors is matched closely by LB low-Re model followed by RKE and then RSM.

4. The match by LB is better along strong Lorentz force bisector.

5. Both RKE and RSM overpredict the MHD sources to TKE along both bisectors.
Low-Re(Re=5602, Ha=21.2): MHD square duct: friction factor

1. As per DNS, along bottom-horizontal wall, the friction factor shows two side peaks with a huge dip at the center.

2. Along left-vertical wall, the friction factor shows a central flat region with two side dips.

3. All the models predicted different profiles along both the walls but failed to match with the DNS.

4. All k-ε models (LB and RKE) give similar profile, with a central overpredicted peak, but matching up to 0.05 distance units from both side walls (or from the corners).

5. Similar to non-MHD duct, RSM predicted side peaks with a central dip along both walls but could not match with the DNS along any wall.

6. RSM suggests maximum frictional losses, especially at the corners.

7. The best match along both walls is achieved by LB model with MHD sources.

Low-Re(Re=5602, Ha=21.2): MHD square duct: pressure gradient

1. LB low-Re model with MHD sources performed best matching within 1% error from DNS predictions.

2. LB model without MHD sources performed next followed by RKE with magnetic induction/electric potential methods.

3. Similar to performance in non-MHD duct, RSM gives highest pressure gradient (being 35% more than DNS).
Summary

- This work tested various turbulence models of k-\(\varepsilon\) and RSM category, currently in use in various industrial applications, for hydrodynamic and MHD turbulence in
  - High and low Reynolds number channel flows
  - Low Reynolds number square duct flows

- In MHD calculations, the MHD sources in k- and \(\varepsilon\)- equations for k-\(\varepsilon\) models and in Reynolds stresses for RSM, as proposed by Kenjereš et al, were implemented through UDFs in the FLUENT.

- The performance of the various models was evaluated based upon their predictions of
  - mean velocities,
  - RMS of velocity fluctuations, turbulent kinetic energy,
  - MHD sources
  - frictional losses

Conclusions: High-Re non-MHD channel flow (Re=45818)

- All the models predicted mean velocity reasonably well.
- TKE is not predicted that well. RSM gives right trends of RMS of velocity fluctuations.
- Usually, errors in predicted TKE and Reynolds stresses increased from the core of the channel towards the wall.
- Clearly, the wall treatment technique is more important than the model in wall-bounded high speed flows.
- Although predictions by SWF/NEWF at a coarse mesh are reasonably well but EWT gives better predictions with all the models when used with \(y+\sim 1\).
Conclusions: Low-Re MHD(Re=4710, Ha=6) and Non-MHD(Re=4586) channel flows

- The SWF and NEWF should not be used in low Reynolds number flows.
- To handle wall in low-Re flows, either use EWT by maintaining y+~1 in the cells next to wall with proper stretching or use low-Re models.
- RKE, SKE, RNG and RSM-linear pressure strain models with EWT performed almost equally well but overpredicted TKE in the core. RSM captured the anisotropy of fluctuations.
- The performance of low-Re k-ε models is better in predicting turbulence than high Re models.
- Although, low-Re RSM-stress omega model predicted TKE better than RSM-linear pressure strain and other k-ε models but missed the anisotropy of Reynolds stresses.
- The LB low-Re k-ε model is found performing better than other models.
- MHD sources showed significant improvements in predictions in low-Re LB k-ε model.
- The improvement by using MHD sources is not much in high Re models (like RKE, SKE and RSM with EWT), perhaps due to not having MHD effect incorporated in wall treatment method.

Conclusions: Low-Re MHD(Re=5602, Ha=21.2) and non-MHD(Re=5466) square duct flows

- As opposed to channel flow (one-wall bounded turbulence), none of the models could predict mean velocities correctly across the whole domain in the square duct.
- Although limited in accuracy, LB is found to be the best in predicting TKE, mean velocity and frictional losses, followed by RKE/SKE with enhanced wall treatment in k-ε category.
- None of k-ε models captured any secondary flows.
- RSM model, although captured secondary flows and anisotropy of Reynolds stresses qualitatively but clearly over predicted the TKE and frictional losses.
- None of the models fully captured the differential effects of the magnetic field on mean velocities, turbulence and frictional losses. The LB with MHD sources captured the differential suppression of TKE closely but missed in the mean velocities.
- RKE/SKE and RSM could not captured magnetic effects in either mean velocity or TKE.
- MHD sources as proposed by Kenjereš et al are found to be performing reasonably well, even in square duct flow, with low-Re LB model where no wall treatment is required.
- With RKE, SKE and RSM models with EWT, they do not show much improvement.
Meshes in different flows

High-Re channel ($Re=45818$) mesh details for high-Re models (domain: 1x1x1), enhanced wall treatment

- 50x10x10 (uniform)
- 80x10x10 (uniform)
- 130x10x10 (uniform)
- 139x10x10 (non-uniform)
  - BL mesh: First cell=0.0089, growth=1.1, rows=20
  - Bell shape, growth=0.56, total counts=139
- 208x10x10 (non-uniform)
  - BL mesh: First cell=0.0089, growth=1.05, rows=20
  - Bell shape, growth=0.6, total count=208
Low-Re channel (Re=4586) mesh details, high-Re models (domain: 1x1x1), enhanced wall treatment

- 50x10x10 (uniform)
- 80x10x10 (uniform)
- 100x10x10 (non-uniform)
  - BL mesh: First cell=0.0055, growth=1.08, rows=10
  - Total counts=100, uniform

- 120x10x10 (non-uniform)
  - BL mesh: First cell=0.004, growth=1.08, rows=10
  - Bell shape, growth=0.51, total count=120

- 158x10x10 (non-uniform)
  - BL mesh: First cell=0.004, growth=1.08, rows=10
  - Bell shape, growth=0.47, total count=158