Large-eddy Simulation of Heat Transfer in Circular Impinging Jet

Bin Zhao

March 22, 2001

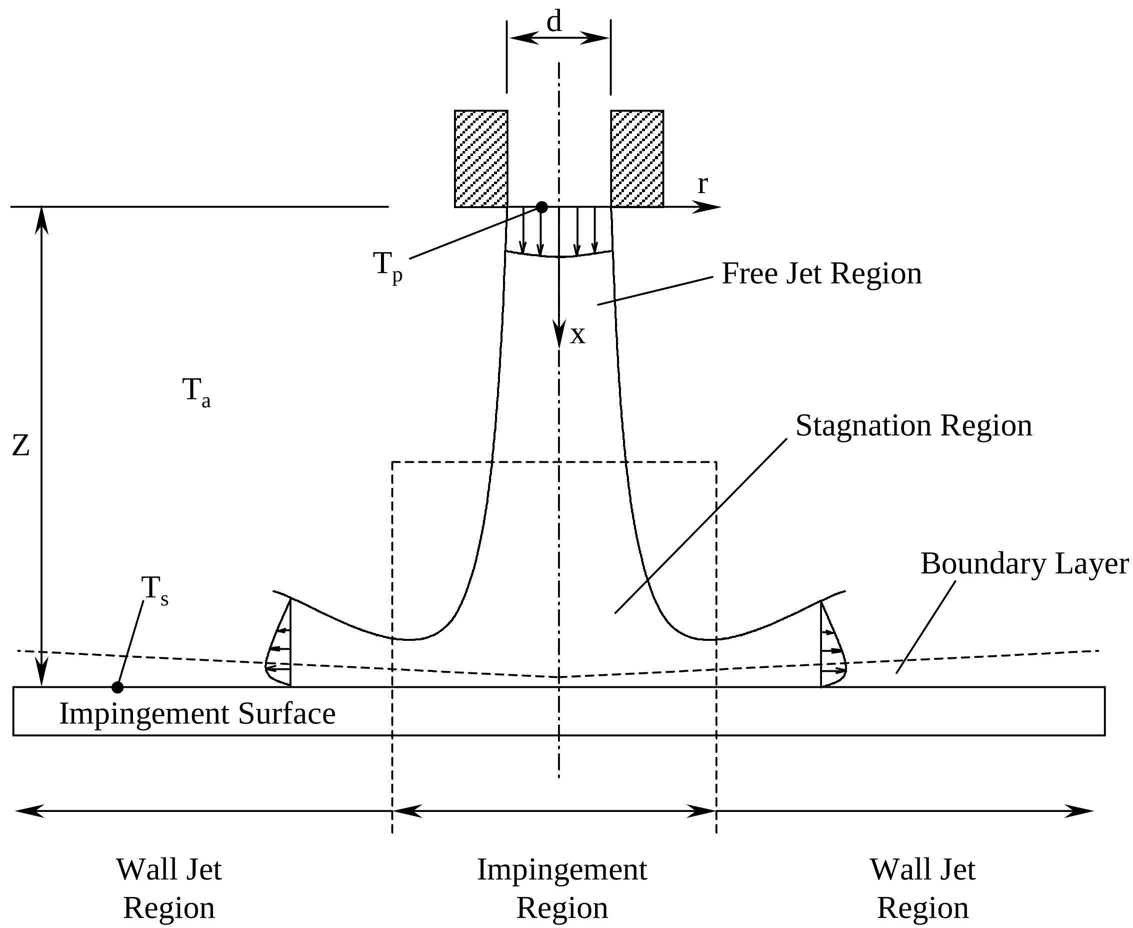
Acknowledgements

- Professor B. G.Thomas & Professor S. P. Vanka
- Accumold
- AK Steel
- Allegheny Ludlum Corp.
- Columbus Steel
- Hatch Associates
- LTV Steel
- Stollberg, Inc
- National Science Foundation (Grant DMI-98-00274)

Introduction

- Several Large Eddy Simulations of the flow and heat transfer in an isothermal circular impinging jet are carried out.
- The experimental conditions of Hollworth and Gero (1985) are simulated to compare the simulations with measurements.

Schematic of the flow region for an impinging jet



Experimental setups

- The experimental case is that an air jet impinging normal to an isothermal flat surface.
- The temperature of the jet is equal to that of the ambient air.
- The target plate is cooled so that its surface temperature is below the ambient temperature.

Experimental Conditions

d	Inlet diameter	$10\,mm$
Z	Nozzle to plate distance	$50\,mm$
T_p	Inlet temperature	$24 \sim 25°C$
T_a	Ambient temperature	T_p
T_s	Impingement surface temperature	$7 \sim 8°C$
ρ	Density of air	$1.2\,Kg/m^3$
μ	Molecule viscosity of air	$17.85\times10^{-6}\ Ns/m^2$
k	Thermal conductivity	$0.25 W/mK$
Pr	Prandtl number	0.71
Re	Renolds number $V_b d/\nu$	5000
V_b	Inlet bulk velocity $\dfrac{4\dot{m}}{\pi d^2}$	$7.4375\,m/s$

Simulations details

- The simulations are carried out for two different jet Renolds numbers (5,000 and 20,000) on two different grids ($256\times64\times32$).
- The Smagorinsky model was used for these simulations.
- A cylindrical mesh with stagger location of velocities and pressures is used.
- The mesh is stretched in the radial direction.
- Instantaneous flow fields from a fully developed turbulent pipe flow simulation are prescribed as the inlet to the computational domain in a time-varying manner.

Governing Equations – LES Flow Model

- Fluid flow

$$\frac{\partial \mathrm{v}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{D\mathrm{v}_i}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\nu_{eff}\left(\frac{\partial \mathrm{v}_i}{\partial x_j} + \frac{\partial \mathrm{v}_j}{\partial x_i}\right) \tag{2}$$

$$\nu_{eff} = \nu_0 + 0.01(\Delta x \Delta y \Delta z)^{2/3}\sqrt{\frac{\partial \mathrm{v}_i}{\partial x_j}\frac{\partial \mathrm{v}_i}{\partial x_j} + \frac{\partial \mathrm{v}_i}{\partial x_j}\frac{\partial \mathrm{v}_j}{\partial x_i}} \tag{3}$$

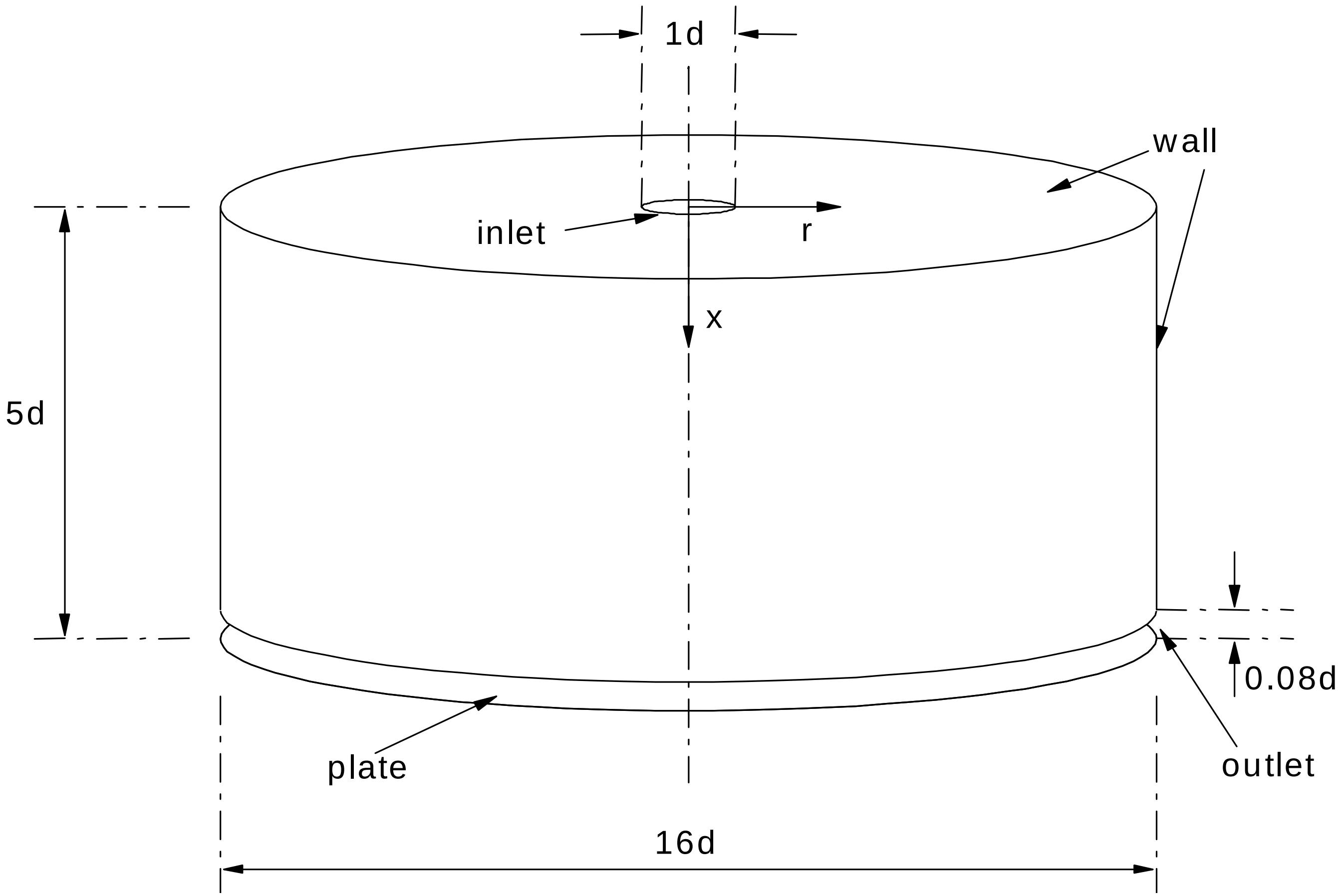
(Smagorinsky sub-grid viscosity model)

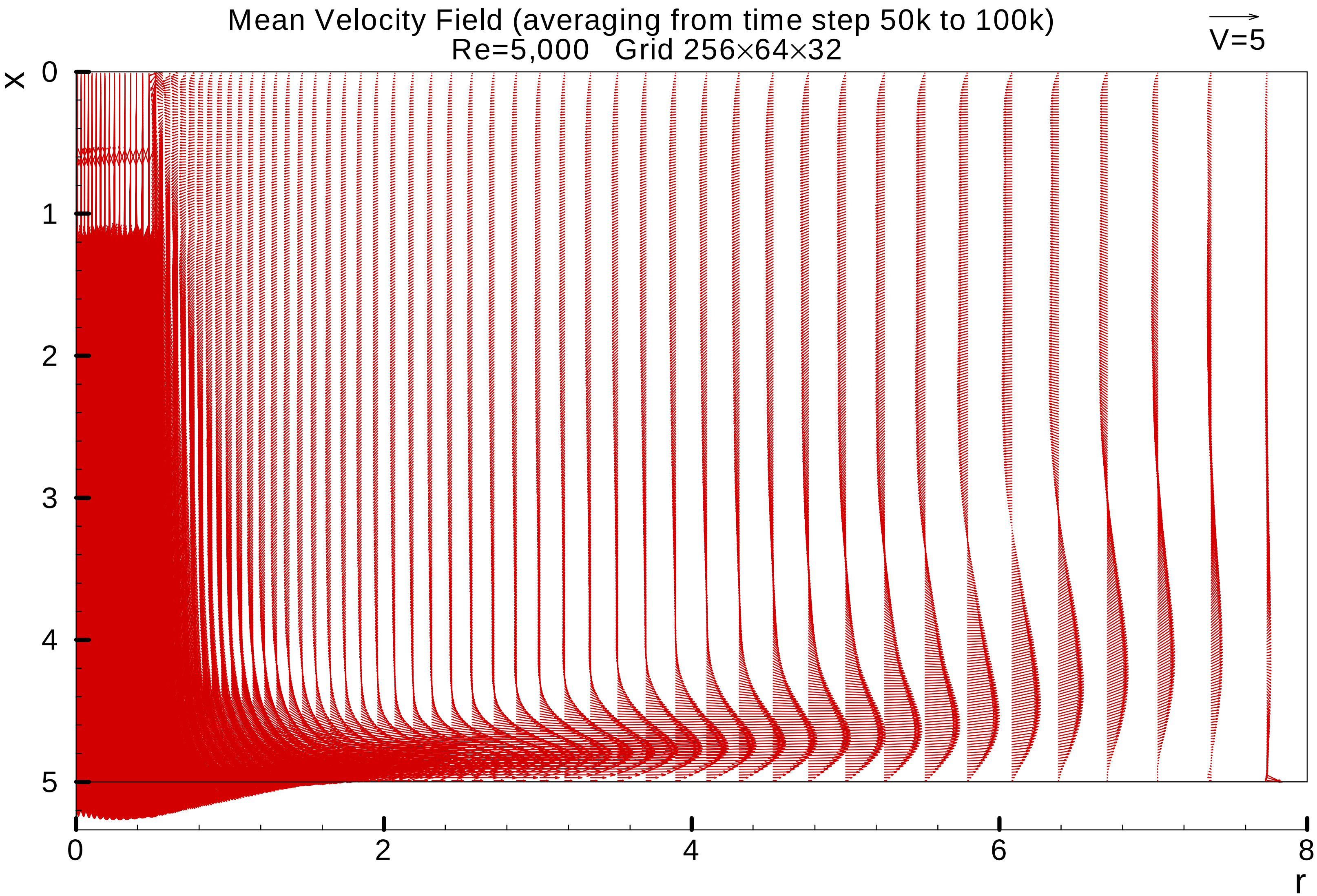
- Energy equation

$$\frac{DT}{Dt} = \frac{\partial}{\partial x_j}\left(\alpha_{eff}\frac{\partial T}{\partial x_j}\right) \tag{4}$$

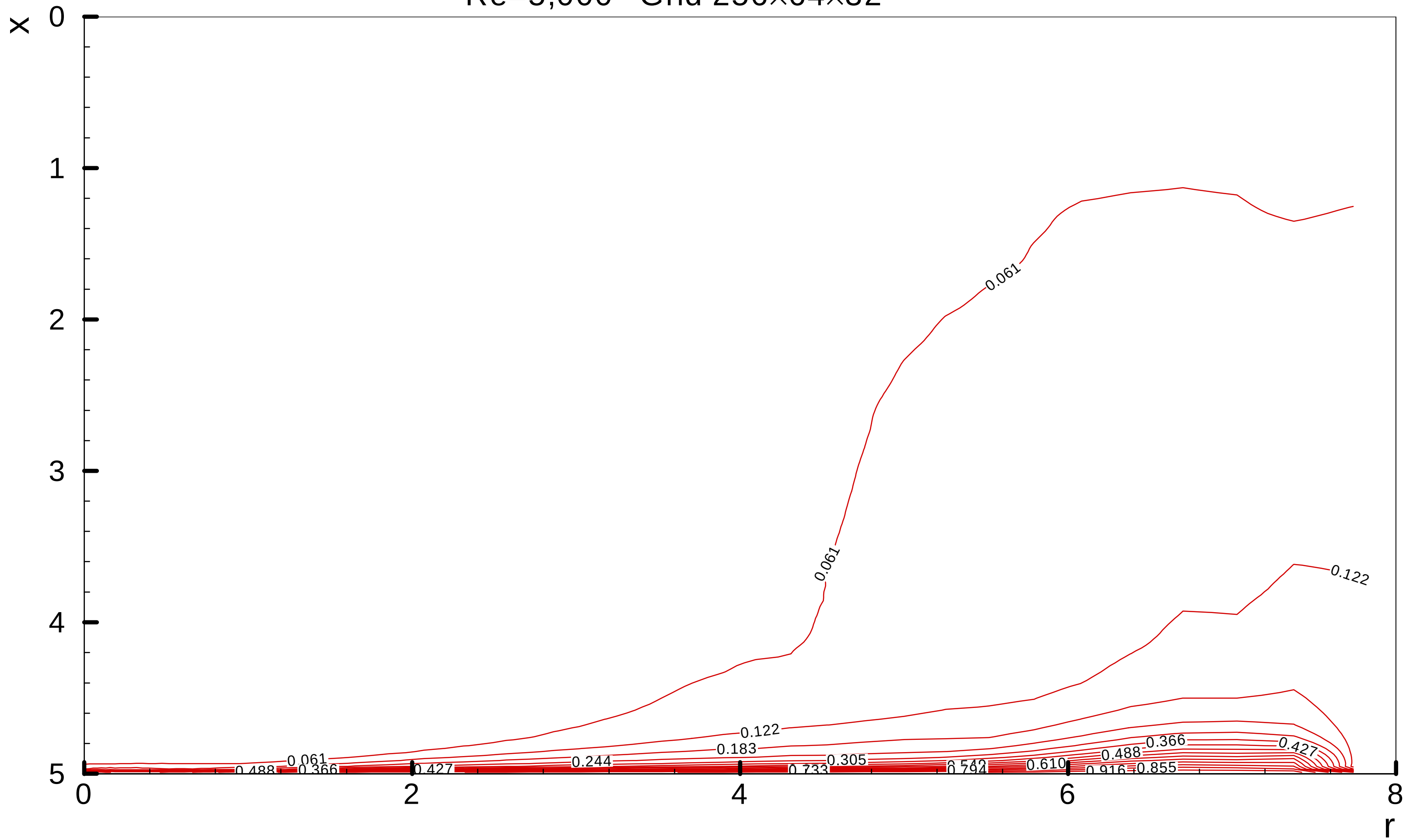
$$\alpha_{eff} = \frac{\nu_0}{\mathrm{Pr}_0} + \frac{\nu_t}{\mathrm{Pr}_t} \tag{5}$$

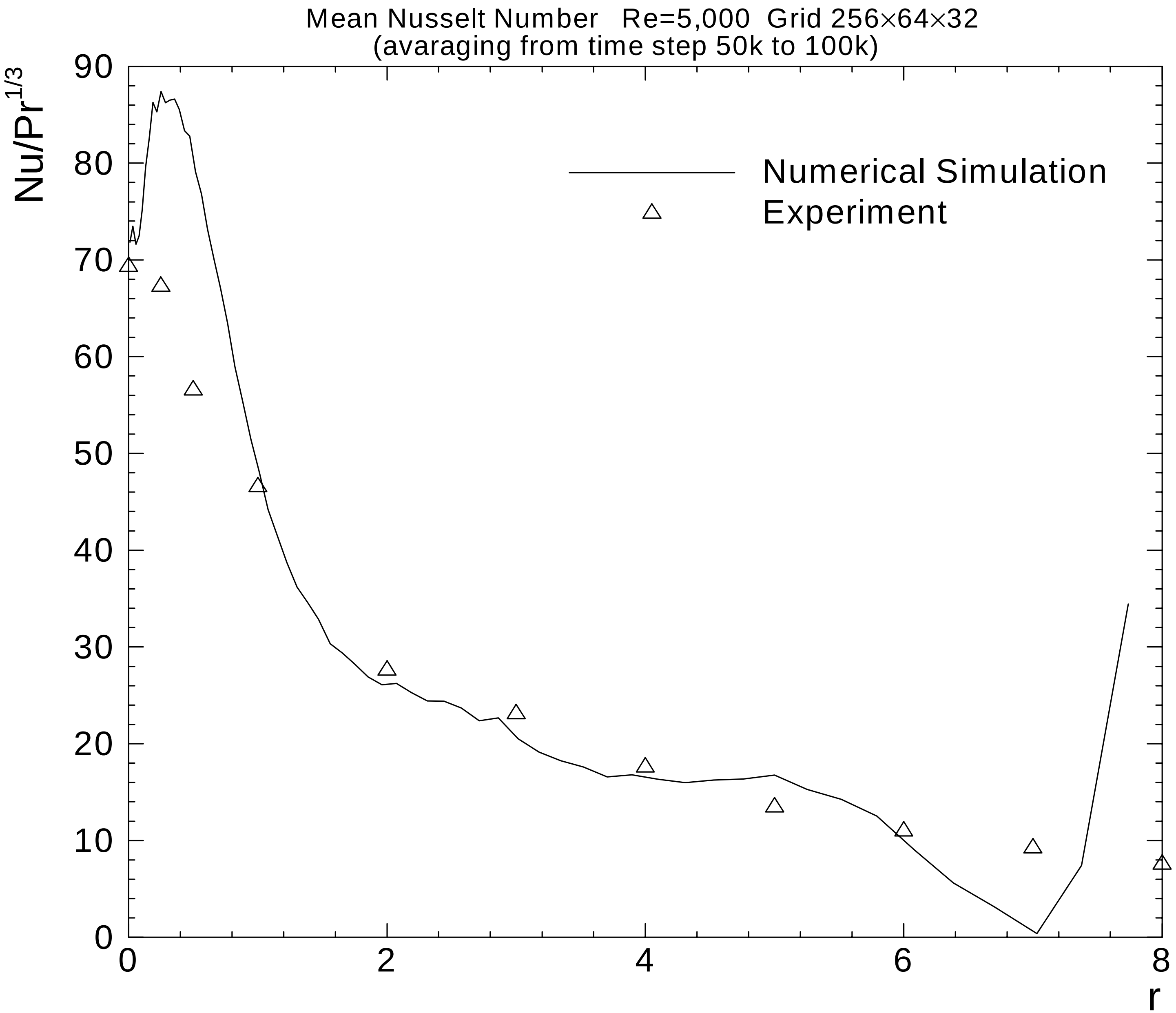
Simulation domain

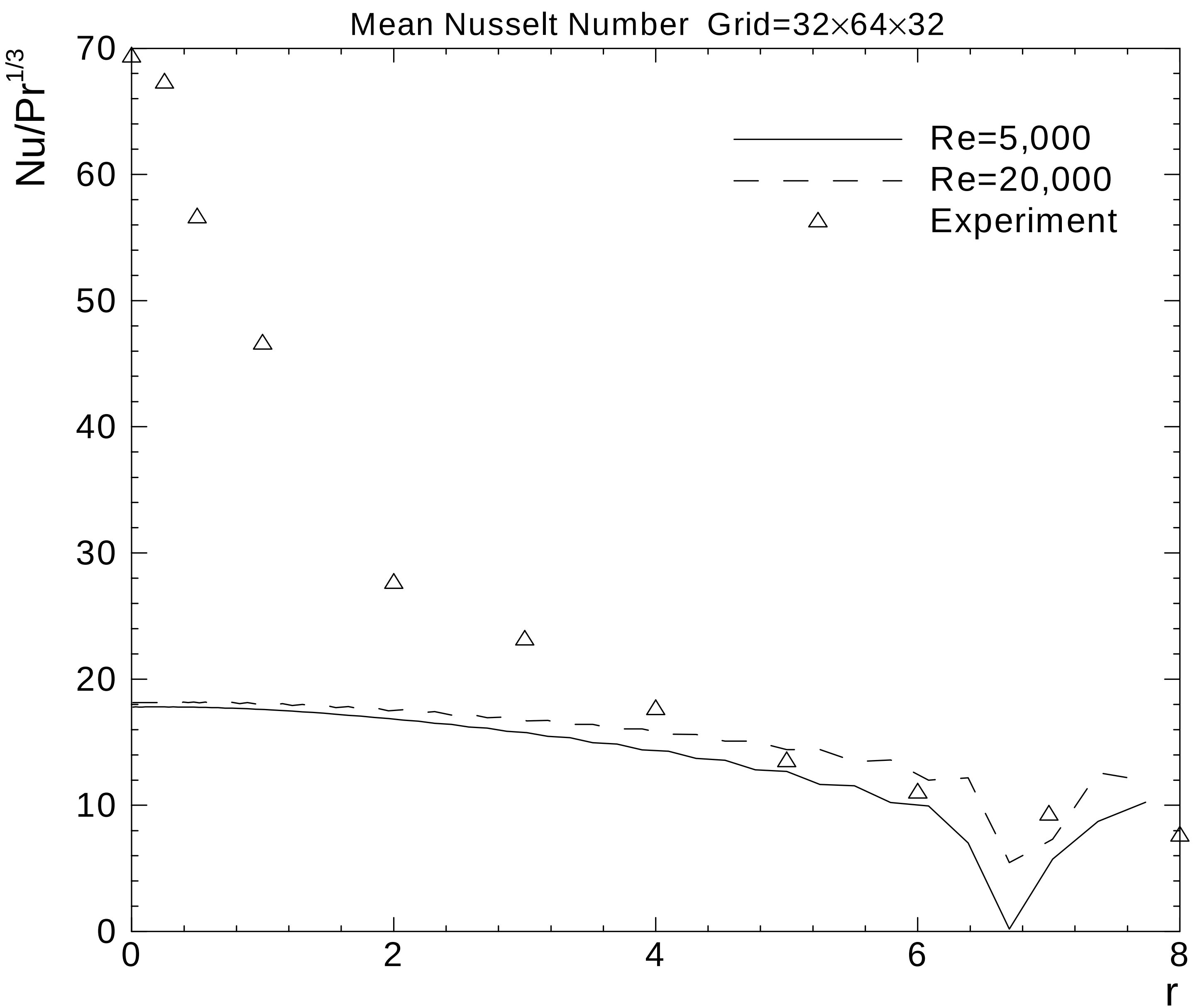



Mean Velocity Field (averaging from time step 50k to 100k)
Re=5,000   Grid 256×64×32
V=5
x
0
1
2
3
4
5
0
2
4
6
8
r

Mean Temperature Contours (averaging from time step 50k to 100k)
Re=5,000 Grid 256×64×32




Mean Nusselt Number Re=5,000 Grid 256×64×32
(avaraging from time step 50k to 100k)
Nu/Pr^1/3
90
80
70
60
50
40
30
20
10
0
0
2
4
6
8
r
Numerical Simulation
Experiment


Mean Nusselt Number Grid=32×64×32
Nu/Pr^1/3
70
60
50
40
30
20
10
0
0
2
4
6
8
r
Re=5,000
Re=20,000
Experiment

Grid Spacing Restriction for Heat Transfer Rate Prediction (I)

The dimensionless temperature is defined as:

$$\theta = \frac{T - T_{inlet}}{T_{inlet} - T_{surface}}$$

The dimensionless length is defined as:

$$x^{*} = \frac{x}{D}$$

Grid Spacing Restriction for Heat Transfer Rate Prediction (II)

The Nusselt number is the dimensionless temperature gradient at the impingement plate. Using a simple first-order differencing scheme, the Nusselt number can be calculated using the following equation:

$$Nu = \left.\frac{\partial \theta}{\partial x^*}\right|_{wall} \approx \left.\frac{\Delta \theta}{\Delta x^*}\right|_{wall}$$

Grid Spacing Restriction for Heat Transfer Rate Prediction (III)

The maximum dimensionless temperature difference:

$$\Delta\theta_{\max} \le 1$$

The maximum Nusselt number:

$$Nu_{\max} \approx \frac{\Delta\theta_{\max}}{\Delta x^{*}} \le \frac{1}{\Delta x^{*}}$$

So, the distance between the first grid point and the plate must satisfy the following restriction in order to predict the heat transfer rate correctly

$$\Delta x^{*} \le \frac{1}{Nu_{\max}}$$

Conclusions

- There is a restriction on the grid spacing to predict the heat transfer rate correctly.
- Finer grids are needed for higher Reynolds number simulations.

Preliminary calculations for continuous casting mold simulation

Empirical equation (I)

$$Nu = K\,\mathrm{Re}^{a}$$

$$a = 0.82 - \frac{0.32}{\left(1 - 1.95\left(\frac{r}{D}\right)^{1.8} + 2.23\left(\frac{r}{D}\right)^{2}\right)\left(1 - 0.21\left(\frac{x}{D}\right)^{1.25} + 0.21\left(\frac{x}{D}\right)^{1.5}\right)}$$

Source: K. Jambunathan, A review of heat transfer data for single circular jet impingement, Int. J. Heat and Fluid Flow, Vol. 13, No. 2

Empirical equation (II)

$$Nu = Ae^{-(B+C\cos\Phi)(r/D)^{0.75}}\ \mathrm{Re}^{0.7}$$

Source: R. J. Goldstein, Heat transfer from a flat surface to an oblique impinging jet, ASME J. Heat Transfer, Vol. 110, 84-90

The Effect of Prandtl Number on Heat Transfer Rate

The two empirical equations are all based on the air impinging jet data, so the Nu number calculated from these equations are Nu numbers for air (Pr = 0.71). While for steel, Pr = 0.1. So the Nu number will be different.

However, the Nu number can be assumed to be proportional to $Pr^{1/3}$*. So we can apply the empirical equations to the steel case using the following equation:

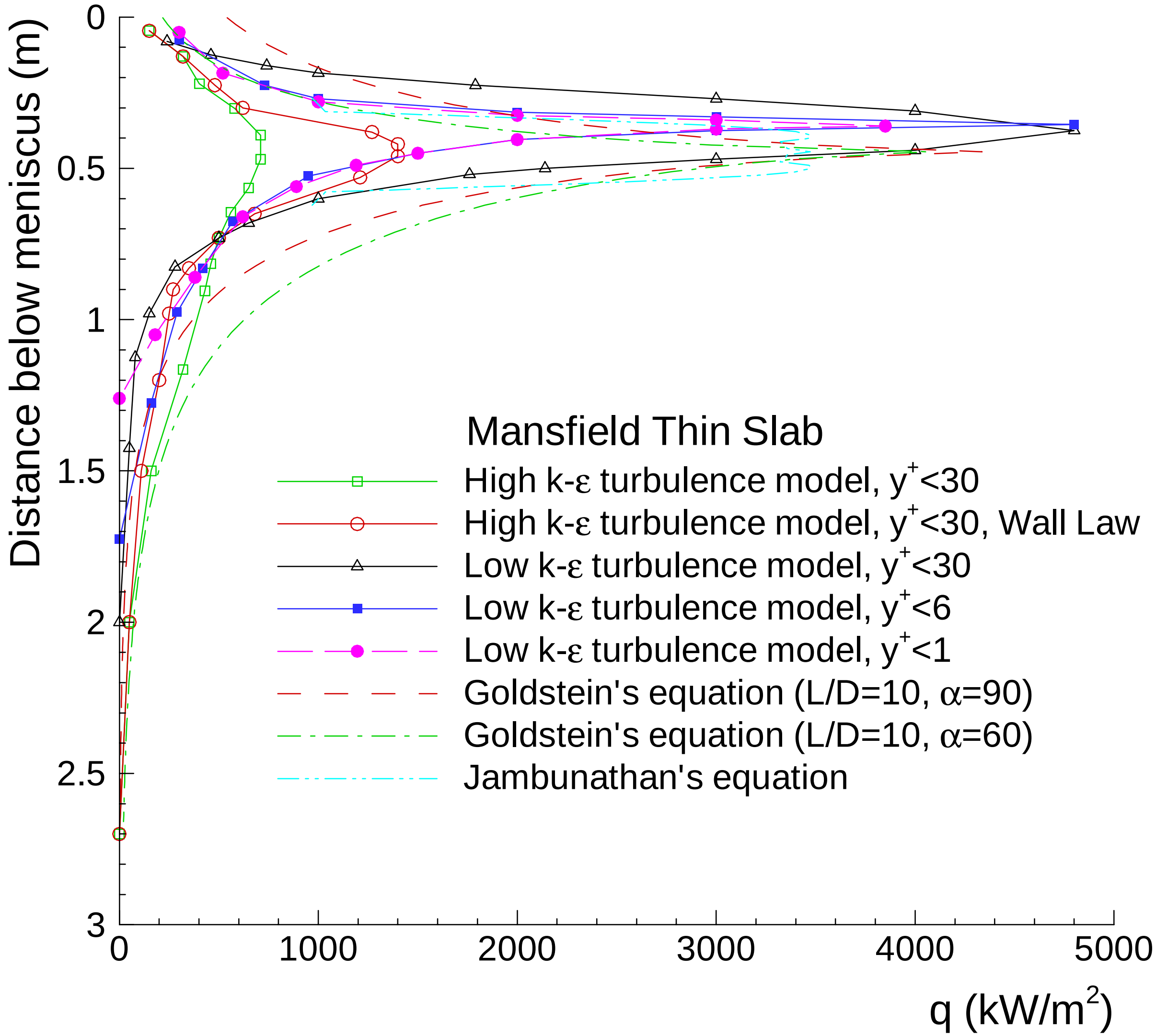
$$Nu \propto \mathrm{Pr}^{1/3} \qquad \frac{Nu_{steel}}{\mathrm{Pr}_{steel}^{1/3}} = \frac{Nu_{air}}{\mathrm{Pr}_{air}^{1/3}}$$

* K. Jambunathan, A review of heat transfer data for single circular jet impingement, Int. J. Heat and Fluid Flow, Vol. 13, No. 2

Casting conditions (Mansfield thin slab)

Casting speed	0.0254 m/s
Mold thickness	0.132 m
Mold width	0.984 m
Inlet height	0.07 m
Inlet width	0.022 m
Nozzle submerge depth	0.126 m
Inlet temperature	1836 K
Liquidus temperature	1775 K
Density	7020 kg/m^3
Thermal conductivity	26 W/mK
Prandtl number	0.1

Heat flux prediction (Mansfield Thin Slab)



Grid restriction (Mansfield Thin Slab)

Maximum Nusselt number:

$$Nu_{max} = 97$$

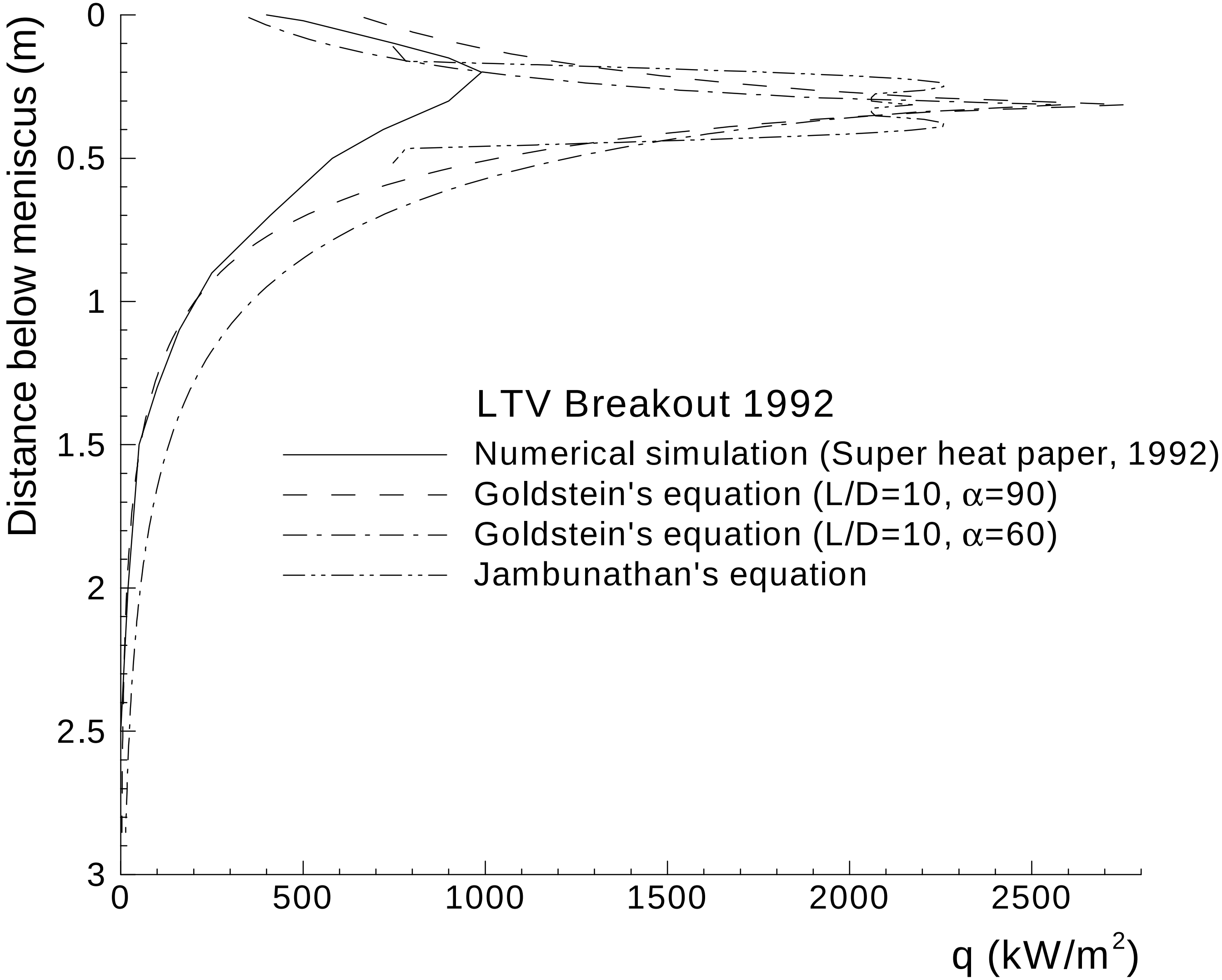
Grid spacing at the impinging plate:

$$\Delta x \leq 0.457 \text{ mm}$$

Casting conditions (LTV Breakout 1992)

Casting speed	0.0267 m/s
Mold thickness	0.22 m
Mold width	1.05 m
Inlet height	0.045 m
Inlet width	0.045 m
Nozzle submerge depth	0.1 m
Inlet temperature	1558 °C
Liquidus temperature	1531 °C
Density	6968 kg/m^3
Thermal conductivity	26 W/mK
Prandtl number	0.1

Heat flux prediction (LTV Breakout 1992)



Grid restriction (LTV Breakout 1992)

Maximum Nusselt number:

$$Nu_{max} = 158$$

Grid spacing at the impinging plate:

$$\Delta x \leq 0.323 \text{ mm}$$